Additional Classical Hall Effect Measurements and Calculations for the Quantum Hall Effect Experiment

Earlier you saw from equation (1) in the classical hall effect primer that:

(1) $$V_{Hall} = -\frac{IB}{nqt}$$

knowing the thickness of your sample is nominally 1mm allows you to calculate the carrier densities of each device. It was mentioned that this can change in semiconductors. We'll define R_H as the hall coefficient:

(2) $$R_H = \frac{1}{qn} \text{ or } -\frac{1}{qp}$$

Where q is the charge of the electron, n is the carrier density of electrons, and p (about to be confused with a Greek letter soon, so keep track) is the carrier density of holes when only one type of charge carrier is present. We'll denote mobility (how fast a charge moves when an electric field acts on it) as either μ_e or μ_h when dealing with electrons or holes. When both carrier types are present, equation (2) becomes:

(3) $$R_H = \frac{p\mu_h^2 - n\mu_e^2}{q(p\mu_h - n\mu_e)^2}$$

Which should reduce to (2) when the carrier density of one type of charge drops to zero. We'll assume for the purpose of the lab that (2) holds, but multiple bands of charge flow in a material can change the numerator of (2) from -.2 to 1.5 or so depending on the material and strength of the applied field (for more information, see Ashcroft and Mermin's *Solid State Physics* as an example). One last definition that we'll need for your measurements:

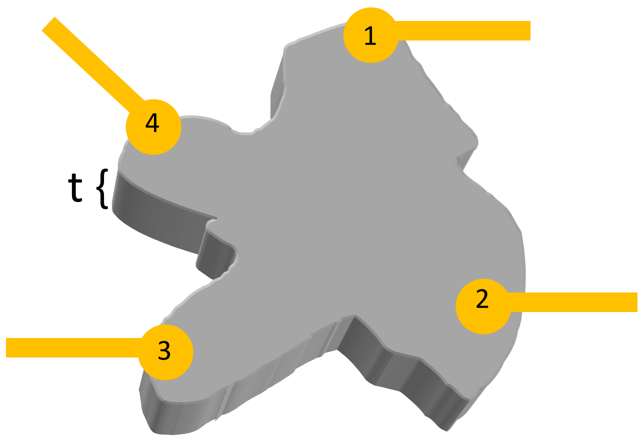
(4) $$\mu_{e/h} = \frac{R_H}{\rho}$$

Where ρ (Greek letter, not lower case p) is the resistivity of the sample. If we know the Hall coefficient and the sample's resistivity, we can determine the sign, density, and mobility of the charge carriers (or vice-versa). To get the resistivity, we need to use the Van der Pauw technique unless our sample is perfectly symmetrical and pure (otherwise the placement of your leads and other factors will give different results). In solid state physics you may see setups referred to as Hall bars. Our sample is a pretty decent rectangle, but you've hopefully noticed a nonzero offset voltage and that the idealistic theoretical model doesn't quite hold.

The Van der Pauw tecnique:

The Theory:

Take any arbitrary shape, so long as it is thin, conducting throughout, and doesn't have any holes. With 4 contacts along the edges you can calculate the sheet resistance R_s for a sample of thickness t at zero applied magnetic field:



You should get two different resistances for two different sets of contacts on the edges of the sample. Definitions for the measurement:

$\boldsymbol{V_{mn}}$ = measured voltage from contact m to contact n

$\boldsymbol{I_{ab}}$ = supplied current from contact a to contact b

We'll make 2 resistance measurements: $\boldsymbol{\Omega_1} = \frac{\mathbf{V_{43}}}{\mathbf{I_{12}}}$ and $\boldsymbol{\Omega_2} = \frac{\mathbf{V_{14}}}{\mathbf{I_{23}}}$, then from the Van der Pauw technique we arrive at:

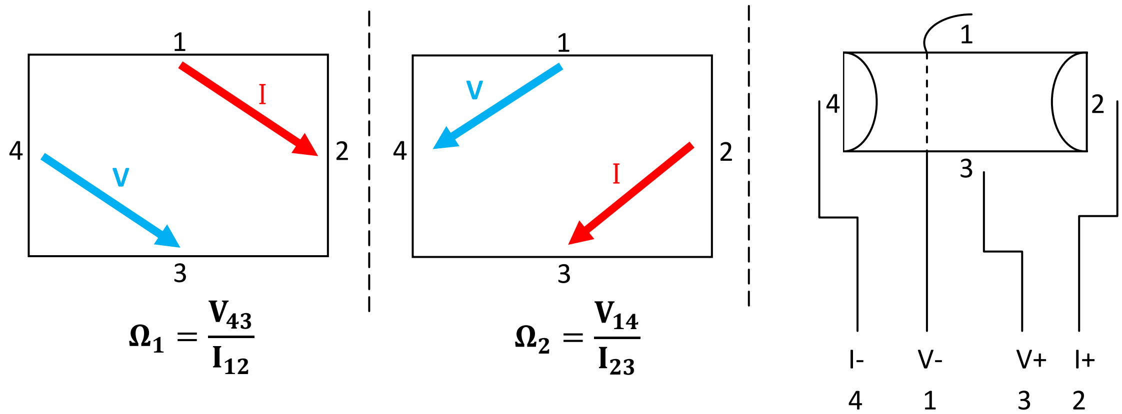
(5) $$e^{-\pi\frac{\Omega_1}{R_s}} + e^{-\pi\frac{\Omega_2}{R_s}} = 1$$

You can solve (5) for the sheet resistance R_s, then the resistivity ρ is simply:

(6) $$\rho = R_s t$$

In EE, you may see this simplified to $\boldsymbol{R_s} = \boldsymbol{f}\frac{\pi R_{VDP}}{\ln 2}$, where $R_{VDP} = \Omega_1 = \Omega_2$ and people working in the field memorizing pi/ln(2) ≈4.53 when they work with nicely engineered Hall bars. The f in front is a function of the ratio of resistances and ranges in value from 1 to 0. We'll work that out on our own soon.

The Procedure:



$$\Omega_1 = \frac{V_{43}}{I_{12}} \qquad \Omega_2 = \frac{V_{14}}{I_{23}}$$

Wire up each sample to find the two resistances. To do this, take steps from 1mA to 10mA (probably best not to exceed 10mA, as you're using the smaller side contacts), then look at the slope of $\frac{\Delta V_{43}}{\Delta I_{12}} = \boldsymbol{\Omega_1}$, for example. From here, you would plug in your two resistance values into equation (5) to obtain R_s or ρ . The punch line: you have to solve equation (5) numerically. In the rare case where $\Omega_1 = \Omega_2$ you can solve for $\boldsymbol{R_s} = \frac{\pi\Omega}{\ln 2}$ but you won't see this in practice. Assuming $R_1 \geq R_2$, you can first make a substitution $\boldsymbol{U} = -\frac{\pi \boldsymbol{R_2}}{\boldsymbol{R_s}}$ and $\boldsymbol{Q} = \frac{\boldsymbol{R_1}}{\boldsymbol{R_2}}$ so that (5) becomes $\boldsymbol{e^{QU} + e^{U} = 1}$, the another substitution $\boldsymbol{V = e^{U}}$ to arrive at:

(7) $$\boldsymbol{V^{Q} + V - 1 = 0}$$

Which you can solve in the software of your choosing. In the original Van der Pauw paper a relationship between some function f(Q) and Q was derived for $R_1 \geq R_2$ such that:

(8) $$\boldsymbol{\cosh\left[\left(\frac{Q-1}{Q+1}\right)\frac{ln(2)}{f(Q)}\right] = \frac{1}{2}e^{\frac{ln(2)}{f(Q)}} \quad or \quad \frac{Q-1}{Q+1} = \frac{f(Q)}{ln(2)}arccosh\left(\frac{e^{\frac{ln(2)}{f(Q)}}}{2}\right)}$$

Knowing the value of Q, a logarithmic plot was available to find the value of f(Q) which should range as a product from 0 to 1 times the ratio of pi/ln(2) mentioned before.

Using your results from the Classical Hall Effect Primer and resistivity measurements here, calculate the mobility of each sample using equation (4) above.

Following along with Van der Pauw's original paper, we can go back and calculate the Hall coefficient for each sample and find the carrier density from equation (2). To do this, we take a Hall voltage measurement at zero field then apply some B field and measure the change in resistance.

(9) $$\boldsymbol{R_H} = \frac{t}{B}\frac{V_{Hall}}{I} = \frac{t}{B}\frac{\Delta V_{mn}}{I_{ab}} = \frac{t}{B}\Delta\Omega_{\mathbf{ab,mn}}$$

The second half of (9) works assuming no offsets. We're going to try and directly measure the Hall voltage and get rid of the offsets. Set up some tables– 1.) for B = –, I = +, 2.) for B = –, I = –, 3.) for B = +, I = +, and 4.) for B = +, I = +. The magnitude of your current measurements should be roughly identical, and the magnitude of the B field should be identical (possibly difficult due to having to move the sample and keep the field steady).

Do for each field measurement- so 2 times in total per sample

I contacts	I measured	V contacts	V measured
13	Do 1–10mA	24	
31	Do 1–10mA	24	
24	Do 1–10mA	13	
42	Do 1–10mA	13	

Swapping the field should help deal with the offset to the Hall voltage generated by geometry of the leads (it's independent of the direction of the field), and swapping the current leads should help deal with the offset due to thermal-electric voltage from temperature (sample heats up regardless of which way current is run through it, but will vary with time). Assuming the mobility of the sample is high enough, equation (1) should look more like the following:

(1a) for B = +, I = +: $\boldsymbol{V_{measured} = V_{Hall} + \alpha R_s I_{measured} + V_{Thermo} = V_a}$

(1b) for B = +, I = –: $\boldsymbol{V_{measured} = -V_{Hall} - \alpha R_s I_{measured} + V_{Thermo} = V_b}$

(1c) for B = –, I = +: $\boldsymbol{V_{measured} = -V_{Hall} + \alpha R_s I_{measured} + V_{Thermo} = V_c}$

(1d) for B = –, I = –: $\boldsymbol{V_{measured} = V_{Hall} - \alpha R_s I_{measured} + V_{Thermo} = V_d}$

Where α is some function that depends on the geometry of the sample and leads.

Now arranging some terms you get:

(10) $$\frac{\boldsymbol{R_H I B}}{\boldsymbol{t}} = \boldsymbol{V_{Hall}} = \frac{[(\boldsymbol{V_a}+\boldsymbol{V_d})-(\boldsymbol{V_b}+\boldsymbol{V_c})]}{\boldsymbol{4}}$$

Use (10) and (2) to arrive at a carrier density for each sample. Compare with your results from the Classical Hall effect primer. Compare your mobility measurements to the last step and report how much (and why) they differ.

Finally, you can follow along with the original Van der Pauw paper and do one last set of tables at zero magnetic field and a constant defined current:

I contacts	I measured	V contacts	V measured	R subscript	R calculated
14		23		1	
41		32		2	
23		14		3	
32		41		4	
12		43		5	
21		34		6	
43		12		7	
34		21		8	

Next, take average resistance values and come up with the following terms:

$$\boldsymbol{R_A = \frac{1}{2}(R_1 + R_2), \quad R_B = \frac{1}{2}(R_3 + R_4), \quad R_C = \frac{1}{2}(R_5 + R_6), \quad R_D = \frac{1}{2}(R_7 + R_8),}$$

To arrive at a set of resistance measurements for equation (5) again:

$$\boldsymbol{\Omega_1 = \frac{1}{2}(\mathrm{R_A} + \mathrm{R_B}) = \frac{1}{4}\sum_{n=1}^{4} \mathrm{R_n} \ \ \text{and} \ \ \Omega_2 = \frac{1}{2}(\mathrm{R_C} + \mathrm{R_D}) = \frac{1}{4}\sum_{n=5}^{8} \mathrm{R_n}}$$

In order, using equations (5), (6), (4), and (2), arrive at a final measurement of carrier density for each sample. Compare with the other 2 methods and describe why they might differ.

Final note: your TA threw this together quickly– so check your units, as there's different carrier densities for a sheet and in total involved. there may be a place where the t is inherently present or absent that I missed).