E.K.A. ADVANCED PHYSICS LABORATORY

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QUANTUM HALL EFFECT: FIRST ORDER CORRECTIONS TO THE QUANTIZATION

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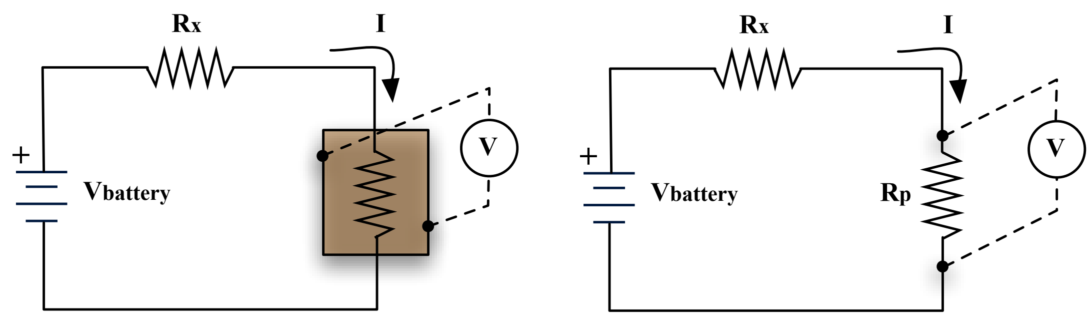

(A) Collecting data (B) Pre-determining the current

FIGURE 1. Circuit configurations

When taking data in the QHE lab, the computer records the Hall resistance R_{Hall} as the measured voltage V over the current I,

$$R_{Hall} = \frac{|V_{Hall}|}{I} \,,$$

as shown in Figure 1a. The current is not measured in real time, but rather calculated in advance by substituting a precision resistor R_p in place of the actual QHE element, shown in Figure 1b. However, the Hall resistance, unlike the precision resistor, varies with the magnetic field. This variation alters the current while the measurement is taking place, which leads to errors in the recorded values of R_{Hall}. You can correct for most of this error using the following procedure.

Each data file is written as a list of coordinate pairs in the form (B,R_{Hall}), where B is the magnetic field and R_{Hall} is the detected voltage over the current,

$$R_{Hall} = \frac{|V_{Hall}|}{I} \,. \tag{1}$$

The current I in the circuit is calculated by measuring the voltage drop across R_p,

$$I = \frac{V_{Rp}}{R_p} = \frac{V_{battery}}{R_x + R_p} \,,$$

yielding (with Equation 1):

$$R_{Hall} = \frac{|V_{Hall}|}{V_{battery}}(R_x + R_p)\ . \tag{2}$$

which are the values recorded by the computer program. During the experiment, the current is actually given by

$$I = \frac{V_{battery}}{R_x + R_{Hall}}\ .$$

The correction can thus be obtained by multiplying both sides of Equation 2 by the factor

$$\left(\frac{R_x + R_{Hall}}{R_x + R_p}\right) \tag{3}$$

yielding the corrected R_{Hall} (denoted by a prime):

$$R'_{Hall} = \frac{|V_{Hall}|}{I_{actual}} = R_{Hall}\left(\frac{R_x + R_{Hall}}{R_x + R_p}\right)\ . \tag{4}$$

In this case the R_{Hall} inside parentheses is approximated by the recorded R_{Hall}, obtained with the predetermined value for I. This correction should be performed for every value of R_{Hall} in every file of data coordinates, keeping in mind the values of R_x may be different for each file.

We would like to know the magnitude of the error $R'_{Hall} - R_{Hall}$ after the correction is applied, and we hope it is smaller than the original error. Note that an exact correction to R_{Hall} would be

$$R'_{Hall} = \frac{|V_{Hall}|}{I_{actual}} = R_{Hall}\left(\frac{R_x + R'_{Hall}}{R_x + R_p}\right)\ , \tag{5}$$

where again the prime indicates the correct value (note the difference with Equation 4). The error in Equation 4 is therefore entirely due to the approximation of the rightmost R'_{Hall} by the known value R_{Hall}. Defining the difference as

$$\Delta R = R'_{Hall} - R_{Hall}\ ,$$

we can rewrite Equation 4 as the exact correction (Equation 5) plus an error term,

$$R'_{Hall} = R_{Hall}\left(\frac{R_x + R'_{Hall} - \Delta R}{R_x + R_p}\right) = R_{Hall}\left(\frac{R_x + R'_{Hall}}{R_x + R_p}\right) - \frac{R_{Hall}\cdot\Delta R}{R_x + R_p}\ . \tag{6}$$

To calculate the size of this error term, we must know ΔR, the error associated with the original value for R_{Hall}. From the approximately correct Equation 4, we can write:

$$\Delta R = R'_{Hall} - R_{Hall} = R_{Hall}\cdot\left(1 - \frac{R_x + R_{Hall}}{R_x + R_p}\right)\ . \tag{7}$$

Consider the value of R_{Hall} at the plateau n=2. Its value is roughly 12,900 ohms, similar to the precision resistor's 12,405 ohms. The highest valued resistor R_x should be the most dependable choice, because it will allow the current to vary the least. The highest R_x used at the time this document was written was about 6×10^7 ohms. Thus, the original error in the recorded R_{Hall} values is roughly

$$R_{Hall}\cdot\left(1 - \frac{R_x + R_{Hall}}{R_x + R_p}\right) \approx 12,900\cdot\left(1 - \frac{10^7 + 12,400}{10^7 + 12,900}\right) = 0.644 \text{ ohms},$$

or as a percentage of the value of R_{Hall},

$$\frac{\Delta R}{R_{Hall}} = \frac{0.644}{12,900} = 5\times10^{-5} = 0.005\%\ ,$$

better than one part in 10,000. This means that if the recorded value is 12,900 ohms, the actual value is within 1 ohm of that number, which is indeed already quite precise. But we hope to do better — humankind's best measurements of the quantization have determined R_{Hall} to better than one part in 10^7. Applying the correction, the residual error is the magnitude of the error term in Equation 6:

$$\frac{R_{Hall} \cdot \Delta R}{R_x + R_p} \approx \frac{12,900 \cdot 0.0005}{10^7 + 12,400} = 6 \times 10^{-7} \text{ ohms},$$

or as a fraction of 12,900,

$$\frac{6 \times 10^{-7}}{12,900} \approx 5 \times 10^{-11} = 0.000000005\% \; .$$

Without a doubt, this number is dwarfed by the other sources of error in the experiment.

How much worse is the case with a higher-numbered plateau, and with the smallest R_x? Taking Equation 7 with $R_x = 10^6$ and $R_{Hall} = 4,300$ ohms (n=6), we have:

$$\Delta R = R_{Hall} \cdot \left(1 - \frac{R_x + R_{Hall}}{R_x + R_p}\right) \approx 4,300 \cdot \left(1 - \frac{10^6 + 4,300}{10^6 + 12,400}\right) = 34.40 \text{ ohms},$$

before the correction, and after,

$$\frac{R_{Hall} \cdot \Delta R}{R_x + R_p} \approx \frac{4,300 \cdot 34.4}{10^6 + 12,400} = 0.146 \text{ ohms},$$

for a percent error of

$$\frac{0.146}{4,300} \approx 3 \times 10^{-5} = 0.003\% \; .$$

After the correction has been applied, it is necessary to investigate the other sources of error. What are they? Do they cause a shift or a scaling in the recorded values of R_{Hall}, or both? Can you estimate their magnitude, and if not, why not? How accurately can you determine a single value of R_{Hall} from the many points that belong to a single plateau? Can the accuracy be improved by incorporating information from the magnetoresistance data? By answering these questions, you should be able to determine if your final answer for R_{Hall} should be derived from a single plateau in a single data run, or as some kind of average of multiple data runs and/or plateau numbers. Your goal for this lab is to report on what techniques you used to calculate this value and its error.