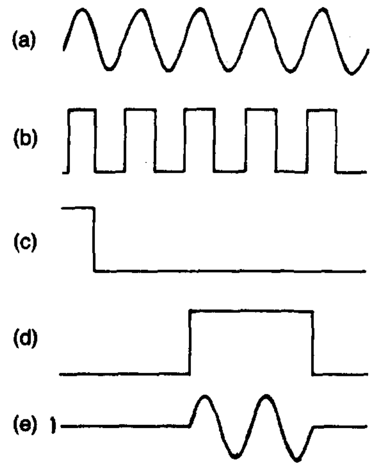


Fig. 2. Representative waveforms for the gating amplifier, observed at (a) CW IN, (b) pin 8 of IC-3A, (c) pin 6 of IC-4, (d) pin 6 of IC-5, and (e) TONE BURST OUT. Amplitudes 2 V (peak to peak) for sinusoids and −5 to +5 V for logic signals. The voltage drop of (c) produces the TRIGGER OUT pulse and the INITIAL DELAY controls determine the later time at which the gate "open" interval of (d) begins.

while the control voltage is "ON." A second analog switch effectively grounds the output at all other times to eliminate undesired feedthrough. At the "Trigger Out" connector, a positive pulse is available with which to synchronize an oscilloscope sweep, to produce a stable display of the entire gated signal. Representative waveforms within the circuit are indicated in Fig. 2.

Our present device provides unity gain and an undistorted, jitter-free output with a 10-kHz sinusoidal input of between 100 mV and 10 V (peak-to-peak) and with a load resistance greater than 5000 Ω. The gate operates without visible distortion of a 2-V (peak-to-peak) signal for frequencies over the range 0.45–800 kHz. A first-stage adjustment permits the dc offset to be zeroed. With the delay and duration controls adjusted such that the gate is switched on and off at zero-crossings of the continuous signal, the switching transients at the output have amplitudes of around 200 mV and durations of less than 100 nsec. Electronic feedthrough depends upon the quality of internal shielding and component placement; with a 10-kHz, 10-V (p–p) signal, we find that when the gate closes the signal at the output (loaded by a 1-MΩ oscilloscope input resistance) drops by more than 80 dB.

As an example of the use of this device, we present in Fig. 3 the measured frequency spectrum of a repeated four-cycle burst of 10-kHz sinusoidal voltage. Waveform analysis was

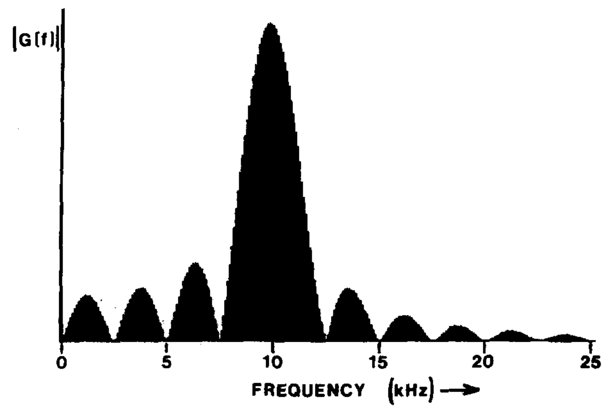


Fig. 3. Measured frequency spectrum of four-cycle bursts of 10-kHz sinusoidal voltage.

performed by a Hewlett–Packard mod. 3581A Wave Analyzer, with 10-Hz bandwidth and 1000-sec linear sweep from dc to 25 kHz. The envelope of this plot agrees well with the magnitude of the Fourier transform for four cycles of a sinusoidal function of frequency $f_0$,

$$G(f) = [Kf_0/(f_0^2 - f^2)] \sin(4\pi f/f_0).$$

We also have found such a gating amplifier useful for study of the transient response of electrical circuits and electroacoustic transducers. For acoustical modelling studies, use of tone bursts permits the separate observation of direct and reflected ultrasonic waves that are propagated in an enclosed space. While some commercially available function generators permit the waveform to be gated, and thus are suitable for these latter applications, we are not aware of any of these more expensive instruments that provides for precise repetition of an arbitrary waveform, with constant burst period, as is required for our Fourier analysis experiment.

Our power supply contains a 25-V rms center-tapped transformer, a bridge rectifier, and solid-state one-chip regulators to provide ±5 and ±8 V. The total cost of parts for the device was less than $100. Further information relative to construction of the gating amplifier will be furnished upon request.[2]

[1]Construction details for an inexpensive wave analyzer are given by P. Ottonello, Am. J. Phys. **45**, 103–104 (1977).

[2]A good general reference for circuit design with CMOS integrated circuits is Don Lancaster, *CMOS Cookbook* (Sams, Indianapolis, 1977).

# Temperature dependence of the platinum resistivity: An experiment for students in solid state and cyrophysics

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## I. INTRODUCTION

We present here a one-week experiment which has been carried out for two years by undergraduate[1] students in physics at the Université Libre de Bruxelles.

We were interested in an experiment to illustrate the lectures in solid state physics and to introduce the students to some of the usual techniques of the physics laboratory,

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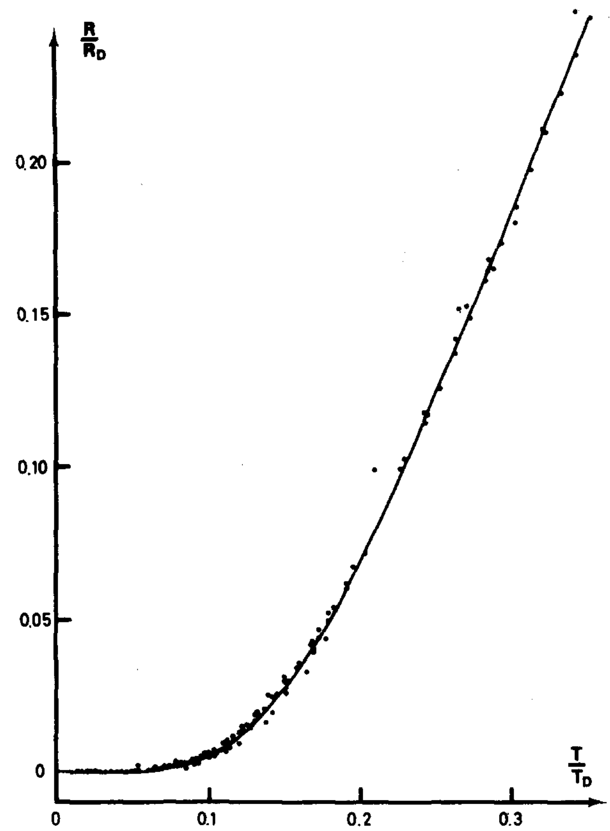


Fig. 1. Experimental values of $r(t)$ obtained by four different students groups compared with the Bloch–Grüneisen relation (2) (continuous line).

viz., liquid nitrogen and liquid helium refrigeration; dc conductivity measurements in the low-voltage region; and data treatment using computer techniques.

The subject we finally selected is the measurement of the electrical resistivity of a metal (platinum) as a function of temperature in the range 2–300K. This experiment, discussed in the framework of phonons and transport theories, is sufficiently simple to concentrate the students' work in the space of one week and sufficiently general to cover the purposes listed above. It is also easily feasible with simple apparatus and apart from a low-temperature facility does not necessitate expensive and sophisticated equipment.

## II. WEEK SCHEDULE

The two first days are used to review the theoretical background, to prepare the experimental procedure and to elaborate a computer program (BASIC) for the data treatment.

The experiment is carried out during the following two days and on the last day, it is suggested to the students to treat their data and to write a short summary and a discussion of their results. For homework, each student is required to prepare a short note on an accessory subject which was discussed with a teaching assistant (e.g., He liquefaction, use of thermocouples, ac measurements, . . . ).

## III. THEORY

The electrical resistivity of metals is due to two mechanisms:

(1) Scattering of electrons on impurities (static imperfections in the lattice).

(2) Scattering of electrons by phonons.

The problem can be simplified by assuming that one scattering process is not influenced by the other (Mathiessen's rule). This rule is not rigorously exact but the errors involved are small.

The first process is usually temperature independent. Its effect on the resistivity is weak compared to the second one, except at very low temperature where the phonon density is vanishing.

The detailed calculation of the electron–phonon scattering contribution to the resistivity, $\rho_{\text{phonons}}$, is rather heavy.[2] No complete analytical solution is possible but an approximate solution is

$$\rho_{\text{phonons}} = \text{const.} \frac{T^5}{\theta^6} \int_0^{\theta/T} \frac{z^5\, dz}{(e^z - 1)(1 - e^{-z})}, \quad (1)$$

which is known as the Bloch–Grüneisen relation.

Apart from the specific scaling parameter $\theta$, the Debye temperature of the metal, the Bloch–Grüneisen relation predicts for all metals a universal temperature dependence of the electrical resistance due to thermal vibrations. Using the reduced variables

$$t = T/\theta,$$

$$r = \frac{R - R_0}{R_D - R_0},$$

where $R$, $R_D$, and $R_0$ are, respectively, the resistance of a given sample at temperature $T$, $\theta$, and zero, it is possible to write for $r$, the relative phonon contribution to the resistance

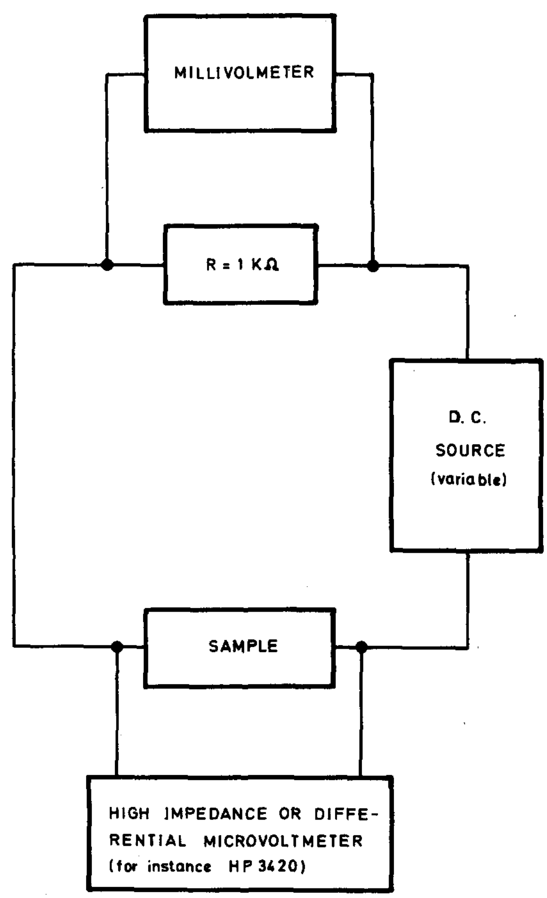


Fig. 2. Four probe circuit used by the students.

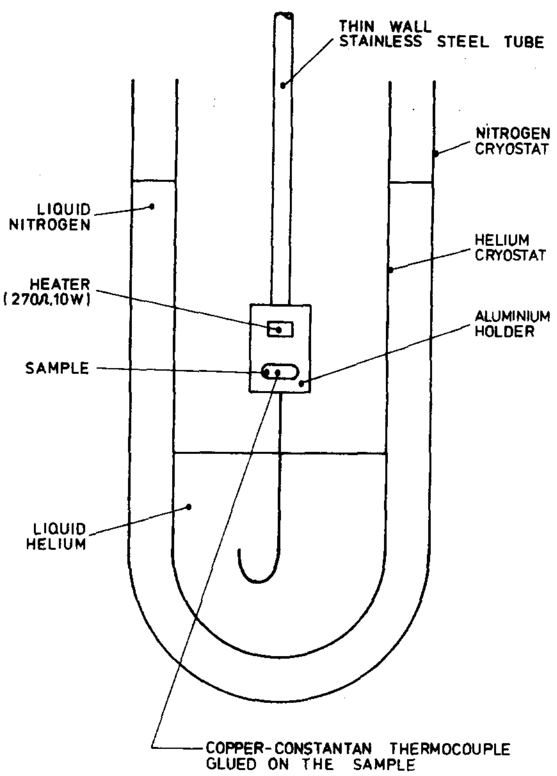


Fig. 3. Arrangement used for the resistivity measurements between 4.2 and 100K.

(normalized to its value at the Debye temperature), the following general form independent of the nature and the size of the metallic sample:

$$r = t^5 \frac{\int_0^{1/t} \left( \frac{z^5 dz}{(e^z - 1)(1 - e^{-z})} \right)}{\int_0^1 \left( \frac{z^5 dz}{(e^z - 1)(1 - e^{-z})} \right)} . \quad (2)$$

This universal function $r(t)$ is represented by a continuous line on Fig. 1. At high temperature ($t > 1/2$) this expression reduces to $r = at$, whereas, at low temperature ($t < 0, 1$) the upper limit of the integral can be taken as infinity and we obtain

$$r = bt^5. \quad (3)$$

For all metals and sample sizes, $a$ is exactly equal to one and $b$, as a result of the computation of the integral, is equal to 497.6.[2]

## IV. MEASUREMENTS

The sample consists of a commercial pure platinum resistance (Oxford Instruments, U.K.) which has a total resistance of about 100 Ω at 273K. The Debye temperature for pure platinum, given in the literature,[3] is 240 K. A four probe circuit is needed to avoid the perturbing effect of the contacts and the probe wires resistances. This circuit is represented on Fig. 2. The sample is attached at the bottom of a thin-wall stainless steel tube located in a homemade helium cryostat surrounded by the usual liquid nitrogen jacket (see Fig. 3). A small copper-constantan thermocouple, glued on the sample, is used to measure the temperature.

Three different techniques are used to carry out the measurements in the range 2–300K:

(1) Between 300 and 100K, the inner dewar is filled with helium gas which provides a thermal link between the sample and the liquid nitrogen jacket. The measurements are taken during the slow cooling down of the sample towards equilibrium (~3 h) without any temperature stabilization.

(2) Between 100 and 4.2K, the sample, in the same dewar, is located above a liquid helium bath. The thermal link is provided by a copper wire (2-mm diameter) attached to the holder and dipping into the helium bath (see Fig. 3). The temperature of the sample is fixed by adjusting the current through the heater, a 270-Ω carbon resistance imbedded in a rectangular brass case screwed to the aluminium holder. Below 20K, the heater is not used and the temperature is modified by variation of the distance between the sample and the helium bath (from 1 to 15 cm).

(3) Below 4.2K, the sample is immersed in the helium bath of which the vapor pressure is regulated to cool the sample down to 2K.[4]

At each temperature, five to six data points of the current–voltage characteristic are taken in the 0–15-mV voltage range.

## V. DATA TREATMENT

The students are asked to write and test a linear regression computer program (least-squares approximation). This program is used first to extract the resistance values from the current-voltage data. Then, the experimental values of $r(t)$ are compared with the Bloch–Grüneisen relation (2). From the low- and high-temperature measurements, respectively, experimental values of $a$ and $b$ are derived, using the program, and compared with the theoretical ones.

## VI. RESULTS

In most cases, the proposed schedule was easily followed and the experimental results appeared reliable for all the student groups. In Fig. 1, the theoretical function $r(t)$ is compared with the experimental data obtained by the different student groups. The results are fairly good. Nevertheless, delicate work is needed to obtain the best agreement and a detailed study of the results shows them to be very sensitive to the experimental ability of the students. This is pointed out in Table I, where the theoretical values of $a$ and $b$ are compared with the experimental values obtained by the different student groups.

The high-temperature coefficient $a$ is always obtained with good precision, keeping in mind that the Bloch–Grüneisen equation is calculated at the limit $T/\theta \rightarrow 0$ and known to differ from the real curve by a maximal amount of 10–15% at the high-temperature limit.[5] On the other

Table I. Theoretical values of $a$ and $b$ [Eqs. (3) and (4)] compared with experimental data obtained by the four students groups (for details and comments, see text and Footnote 6).

| | theory | group 1 | group 2 | group 3 | group 4 |
|---|---|---|---|---|---|
| $a$ | 1 | 1.14 | 1.09 | 1.06 | 1.12 |
| $b$ | 497.6 | 516.5 | 628.2 | 594.6 | 638.2 |

hand, the students experimental values of the low-temperature parameter $b$ differ from the theoretical[6] one by a quantity varying from 3.5 to 28%. This can be easily understood by considering the increased experimental difficulties which occur in the low-temperature range. Indeed, between 4 and 20K, the residual component of the resistance, due to the impurities, is 4–1000 times larger than the thermal contribution. An accurate determination of $b$ requires thus a relative accuracy in the measurements of the order of $10^{-3}$ together with a very precise measure of the absolute temperature. Considering the fact that the temperature is not electronically regulated, a compromise must be found between the time spent for a measurement and its accuracy.

## VII. CONCLUSIONS

This experiment has now been carried out for two years, and has amply fulfilled the goals we set out to achieve. In one week, the students have the opportunity to work on a direct illustration of the lectures on solid state physics, and to get acquainted with the usual low-temperature techniques, the dc conductivity measurements and elementary data processing.

The experiment does not need sophisticated techniques and appears easy to understand in a few hours. Nevertheless, real experimental difficulties exist and the quality of the results depends on the students ability. On the other hand, the preparation of the experiment is short and the material used does not exceed the current possibilities of a solid state physics laboratory.

Occasionally, other samples have been tested. A gold thin film obtained by evaporation under vacuum has given relatively good results. For such a sample, however, the residual resistance prevails in a large temperature range so that the experimental difficulties were greatly increased.


## ACKNOWLEDGMENT

We would like to thank H. Stern for a critical reading of the manuscript.


[1]Third and fourth years of university degree.

[2]The original calculation was derived by F. Bloch, Z. Phys. **59**, 208 (1930); E. Grüneisen, Ann. Phys. **16**, 530 (1933); and A. H. Wilson, Proc. Cambridge Philos. Soc. **33**, 371 (1937). Reference may also be made to Z. M. Ziman, *Electrons and Phonons* (Clarendon, Oxford, 1960). A semi-intuitive derivation of the extreme cases [Eqs. (3) and (4)] can be found in many textbooks like N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Holt, Rinehart and Winston, New York, 1976); R. J. Elliot and A. F. Gibson, *An Introduction to Solid State Physics and its Applications* (Macmillan, London, 1976); and H. M. Rosenberg, *Low Temperature Solid State Physics* (Clarendon, Oxford, 1965).

[3]C. Kittel, *Introduction to Solid State Physics* (Wiley, New York, 1971).

[4]G. K. White, *Experimental Techniques in Low-Temperature Physics* (Clarendon, Oxford, 1979).

[5]D. K. C. Macdonald, International Solvay Institute Reports, 10th Meeting, edited by R. Stoops, Brussels, 1955 (unpublished); E. H. Sondheimer, Proc. R. Soc. London A **203**, 75 (1950).

[6]Note that platinum is not expected to exactly obey Eq. (3). Owing to collisions between $s$ and $d$ electrons (which change the momentum) an additional contribution to the resistance appears. This additional resistance, proportional to $T^2$, is of an order of magnitude which makes it scarcely observable in our experiments. Nevertheless, this $T^2$ contribution is probably responsible for the apparently systematic differences between the theoretical $b$ value and the experimental data reported in Table I. For details on this point see J. L. Olsen, *Electron Transport in Metals*, Interscience tracts on physics and astronomy, edited by R. E. Marshak (Interscience and Wiley, New York, 1962).

# Thermal spring: An attempt to store heat as strain energy


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Sometimes an idea which does not work is more instructive than one which does. Generations of physics students who have learned thermodynamics while attempting to defeat or circumvent the second law are a case in point. An up-to-date challenge is the *thermal spring*, a potential method of storing solar energy based upon the observation that thermal stresses in excess of 50 MPa develop in a steel bar with fixed ends when it is left outside in the sun.

The purpose of this note is to show that, whatever the configuration of the thermal spring, its efficiency is limited by its material properties to impractical values.

The thermal strain in a short, unconstrained, prismatic bar is given by $\epsilon = \alpha(T_2 - T_1)$, where $\alpha$ is the coefficient of thermal expansion. Efficient recovery of the stored energy requires that the maximum stress in the material be less than the yield stress $\sigma_y$. The energy stored in the thermal spring is equal to the work done in restoring the cylinder to its original length:

$$U = \int F\,dx = \int(\sigma A)\,d(\epsilon L) = V\int \sigma\,d\epsilon, \qquad (1)$$

where $A$ is the cross-sectional area, $L$ is the length, and $V$

Table I. Efficiencies for thermal springs made from various common substances.

| Substance | Efficiency $U_{max}/Q$ |
|---|---|
| Reinforced phenolic | 0.0098 |
| High-strength steel | 0.0028 |
| 7075-T6 aluminum | 0.0022 |
| AZ80A magnesium alloy | 0.0019 |
| Hard rubber | 0.0014 |
| Ductile iron | 0.00088 |
| Wood (oak) | 0.00048 |
| Construction steel | 0.00047 |
| Glass | 0.00006 |



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