Introduction:

This experiment touches on the subject of spatial frequency content of objects and how they could be used to control the shape and quality of an image. This involves the application of Fourier Transform which maps intensities to spatial frequencies.

Theory:

Fourier theory states that any signal, in our case visual images, can be expressed as a sum of a series of sinusoids. In the case of imagery, these are sinusoidal variations in brightness across the image. For example the sinusoidal pattern shown below in Figure1 can be captured in a single Fourier term that encodes the spatial frequency, the magnitude and the phase. These three values capture all of the information in the sinusoidal image. The spatial frequency is the frequency across space with which the brightness modulates. For example the image Figure 1(b) shows another sinusoid with a higher spatial frequency.

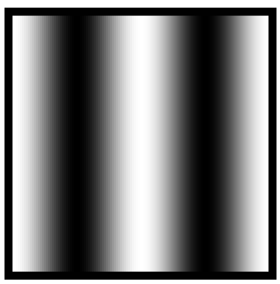

Figure 1.a Lower Spatial Frequency

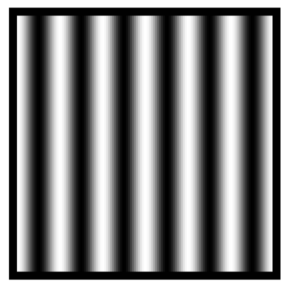

Figure 1.b Higher Spatial Frequency

A Fourier transform encodes not just a single sinusoid, but a whole series of sinusoids through a range of spatial frequencies from zero all the way up to the "nyquist frequency", i.e. the highest spatial frequency that can be encoded in the digital image, which is related to the resolution, or size of the pixels. According to Abbe's theory of imaging, if we place an image, at the focal length of the lens, and illuminate that slide, the lens will automatically perform a Fourier transform on the input image, and project it onto the frosted glass screen as seen in Figure 3.

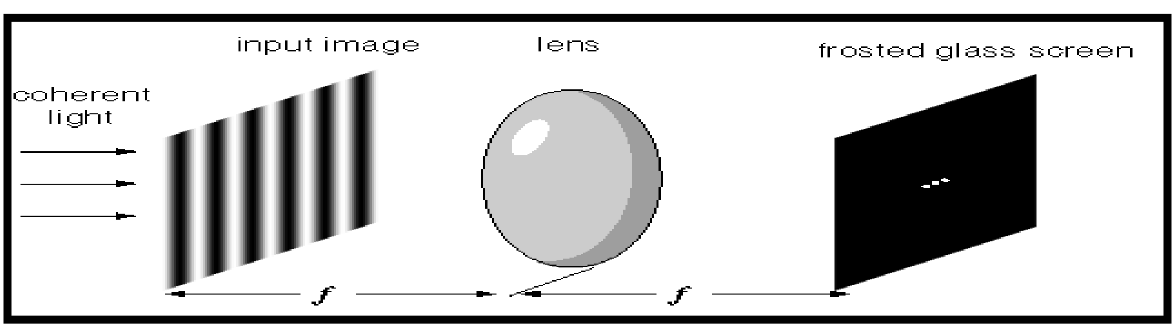

Figure 2: Lens acting as a Fourier Transform lens

Every point on the input image radiates an expanding cone of rays towards the lens, but since the image is at the focus of the lens, those rays will be refracted into a parallel beam that illuminates the entire image at the ground-glass screen. In other words, every point of the input image is spread uniformly over the Fourier image, where constructive and destructive interference will automatically produce the proper Fourier representation.

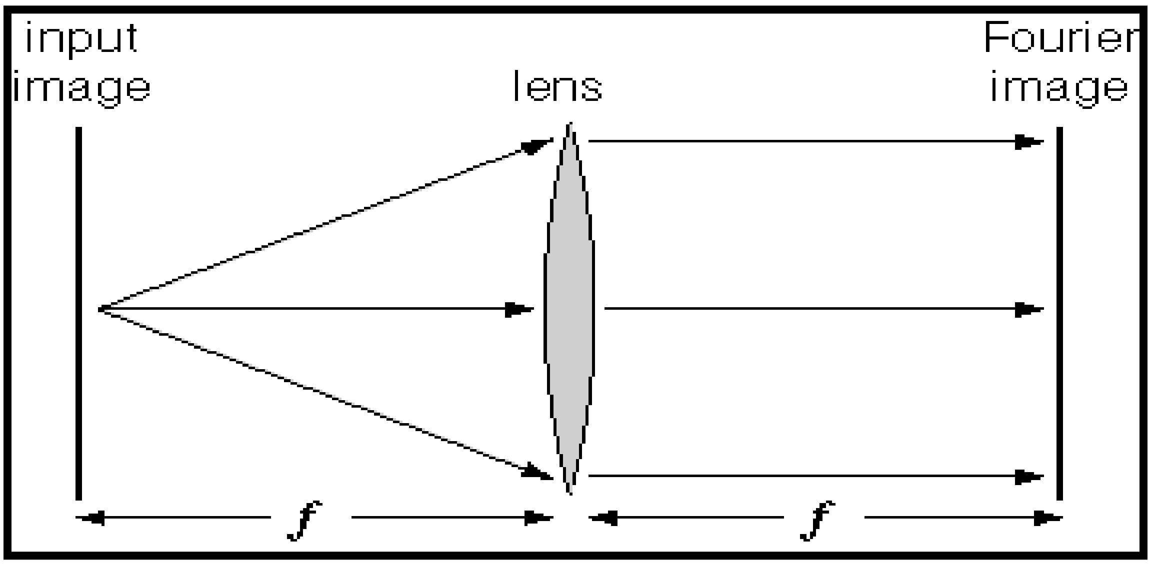

Figure 3: Lens acting as Fourier transform lens

Conversely, parallel rays from the entire input image are focused onto the single central point of the Fourier image, where it defines the central DC term by the average brightness of the input image. Note that the optical Fourier transformer automatically operates in the reverse direction also, where it performs an inverse Fourier transform, converting the Fourier representation back into a spatial brightness image. Mathematically the forward and inverse transforms are identical except for a minus sign that reverses the direction of the computation.

This experiment is similar to finding the frequency harmonic content of a waveform such as that produced by a musical instrument. For example, a musical instrument may produce both a low pitch tone and a high pitch tone. We can control the quality of the sound by filtering out one of the two frequency harmonics with a low pass or a high pass filter. Objects have certain intensity profiles which translate into a corresponding spatial frequency distribution. In this project the frequency distribution of an object illuminated with a laser beam will be examined with a single lens placed after the picture or slide. The light distribution formed at that focal plane tells us the frequency content of the object, and by manipulating the light in that plane we gain control of the quality and content of the image to be displayed. For example, we begin with an input image shown in Figure 5 and perform a Fourier transform on it, and then we do an inverse transform to reconstruct the original image. This reconstructed image is identical, pixel-for-pixel, with the original brightness image.

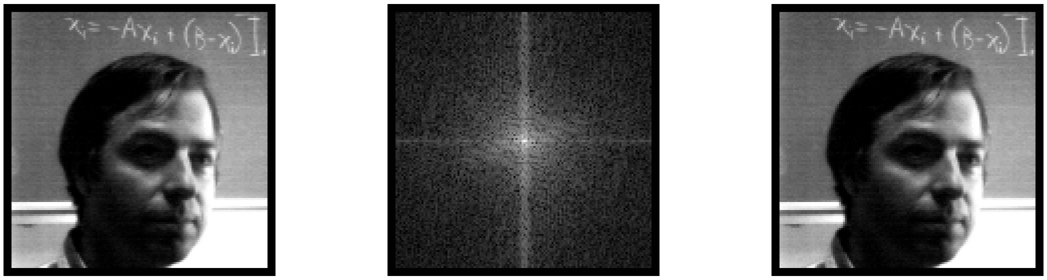

Figure 5: Fourier and Image reconstruction Using inverse Fourier transform

We can manipulate the transformed image to adjust its spatial frequency content, and then perform an inverse transform to produce the Fourier filtered image. For example, if we begin with a low-pass filter, i.e. a filter that allows the low spatial-frequency components to pass through, but cuts off the high spatial frequencies. In other words the low-pass filtered transform is identical to the central portion of the Fourier transform, with the rest of the Fourier image set to zero. An inverse Fourier transform applied to this low-pass filtered image produces the inverse transformed image shown in Figure 6.

Figure 6: Low pass filtered inverse Fourier transform

We see that the low-pass filtered image is blurred, preserving the low frequency broad smooth regions of dark and bright, but losing the sharp contours and crisp edges. Mathematically, low-pass filtering is equivalent to an optical blurring function.

<u>Experimental Set-up:</u>

1) We mounted a laser assembly (LA) to the far side of the breadboard and adjusted the position of the laser such that the beam is parallel to the edge and in line with a line of tapped holes in the breadboard top.

2) We mounted a beam steering assembly (BSA-I) approximately 4 inches in from the far corner of the breadboard and adjusted the height of the mirror mount until the beam intersects the center of the mirror. Then the post in the post holder was rotated until from the laser beam was parallel to the left edge and the surface of the optical breadboard.

3) A second beam steering assembly (BSA-I) was placed in line with the laser beam at the lower left comer of the optical breadboard. The mirror mount was rotated and adjusted until the laser beam is parallel to the front edge and the surface of the optical breadboard.

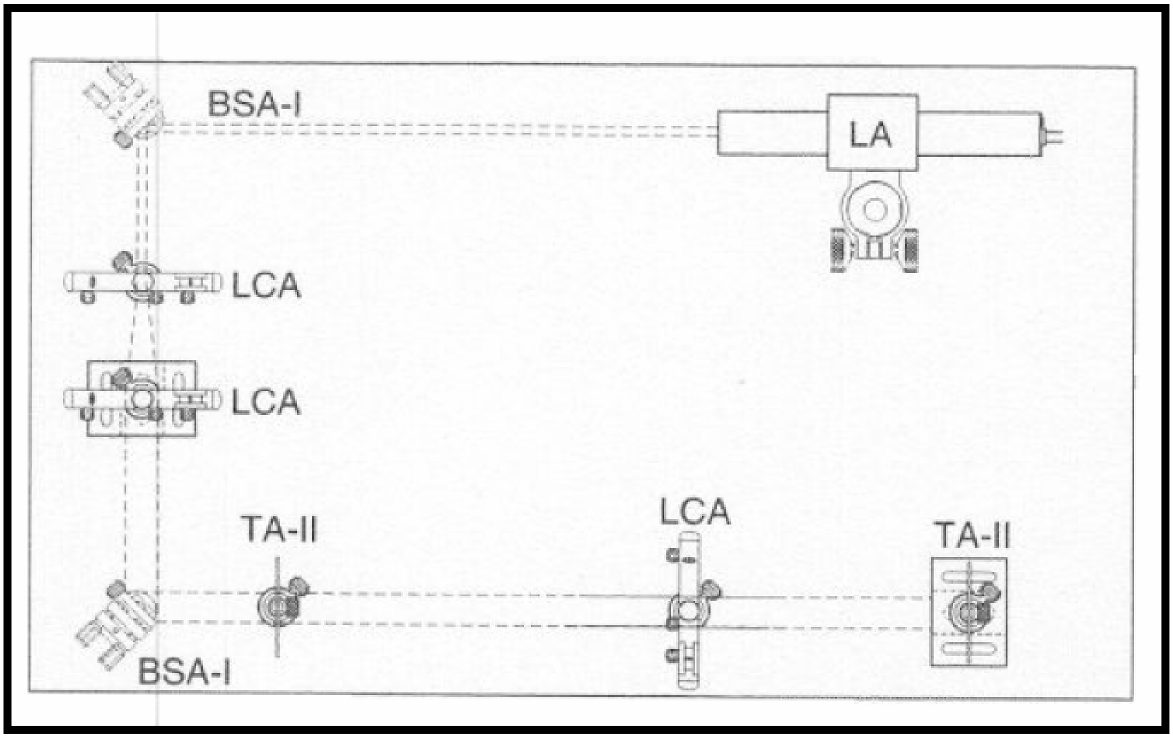

Figure 7: Schematic Diagram of the Imaging Experiment

4) Set up a beam expander between the first two BSA-I's Mount the first lens to the optical table without a B-2 base.

5) We mounted an l5Omm EFL transform lens in an LCA without a base 12 inches from the second BSA and center it in the path of the beam. Set up a modified target assembly with an index card and placed it at the beam focus. 150 mm from the transform lens. The plane of the card represents the back focal plane of the lens.

6) A second modified target assembly was mounted without a base 225 mm before the transform lens. This assembly will be used to hold the picture slides before the transform lens in the path of the laser light. This assembly will be called the slide holder. The set up is now ready for the examination of slides.

7) We placed the target containing the square mesh in the slide holder and replaced the index card to the back focal plane of the lens. There you will see a square grid pattern of dots representing the frequency content of the mesh in both horizontal and the vertical axes. Mark on the card with a pencil the location of these axes. The dots on the x (or y) axis represent frequencies present in that direction in the slide. This image can be manipulated by eliminating certain frequencies, much in the same way a high fidelity

audio filter controls the tone of a musical instrument. To illustrate this, we cut out a narrow vertical slit in a section of the index card such that only those dots on the horizontal axis are passed through the cutout. We noted that the image consists of only horizontal lines. When we remove the slit in the focal plane, the image resembled the original object. We made another cut out such that only the central spot is transmitted. It was noticed that only uniform illumination is present at the observation plane. This is the principle of the spatial filter. It "cleans" optical beams by removing the high frequencies by focusing the beam through a pinhole, thereby obstructing the unwanted harmonics.

Creativity:

We replaced the square mesh slide at the slide holder with a picture slide. At the back focal plane of the lens the light distribution consists of an irregular distribution of light representing the many spatial frequencies present in the picture. Superimposed on this distribution is a set of faint spots aligned along the vertical axis and passing through the central point. This line of spots represents the frequency content of the horizontal lines in the picture slide. Attach with tape to the microscope slide two pins so that when it is placed back in the back focal plane the objects will obstruct this vertical line of spots. Other cutouts may be made and used at the back focal plane. For example, we cut a hole in the index card so that it obstructs the outer spots. These spots contain the high frequency information. Obstructing those spots lead to a softer, fuzzier, picture.

Conclusion:

Thus, this experiment introduces spatial filtering and shows how by changing the spatial frequency content of objects, we could control the shape and quality of an image.