# Biophotonics

## Lecture 3:

Statistical optics

Quantum theory of light

Wei Min

Department of Chemistry

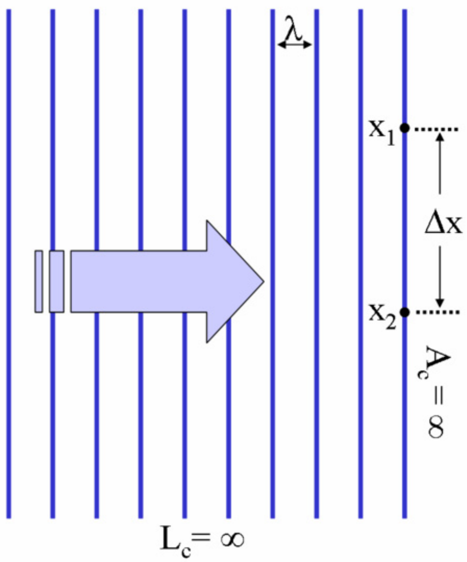
# Outline

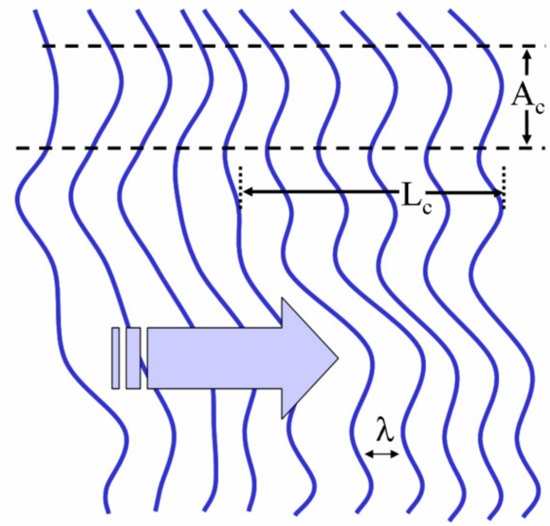
Spatial and temporal coherence

Energy and momentum of photons

Photon counting statistics

Probability wave


λ
x1
Δx
x2
Ac = ∞
Lc= ∞


Ac
Lc
λ

# What is coherence ?

In optics, coherence is a property of waves that enables stationary interference.

Temporal coherence is the measure of the average correlation between the value of a wave at any pair of times, separated by delay

Spatial coherence describes the ability for two points in space, *x1* and *x2*, in the extent of a wave to interfere, when averaged over time.

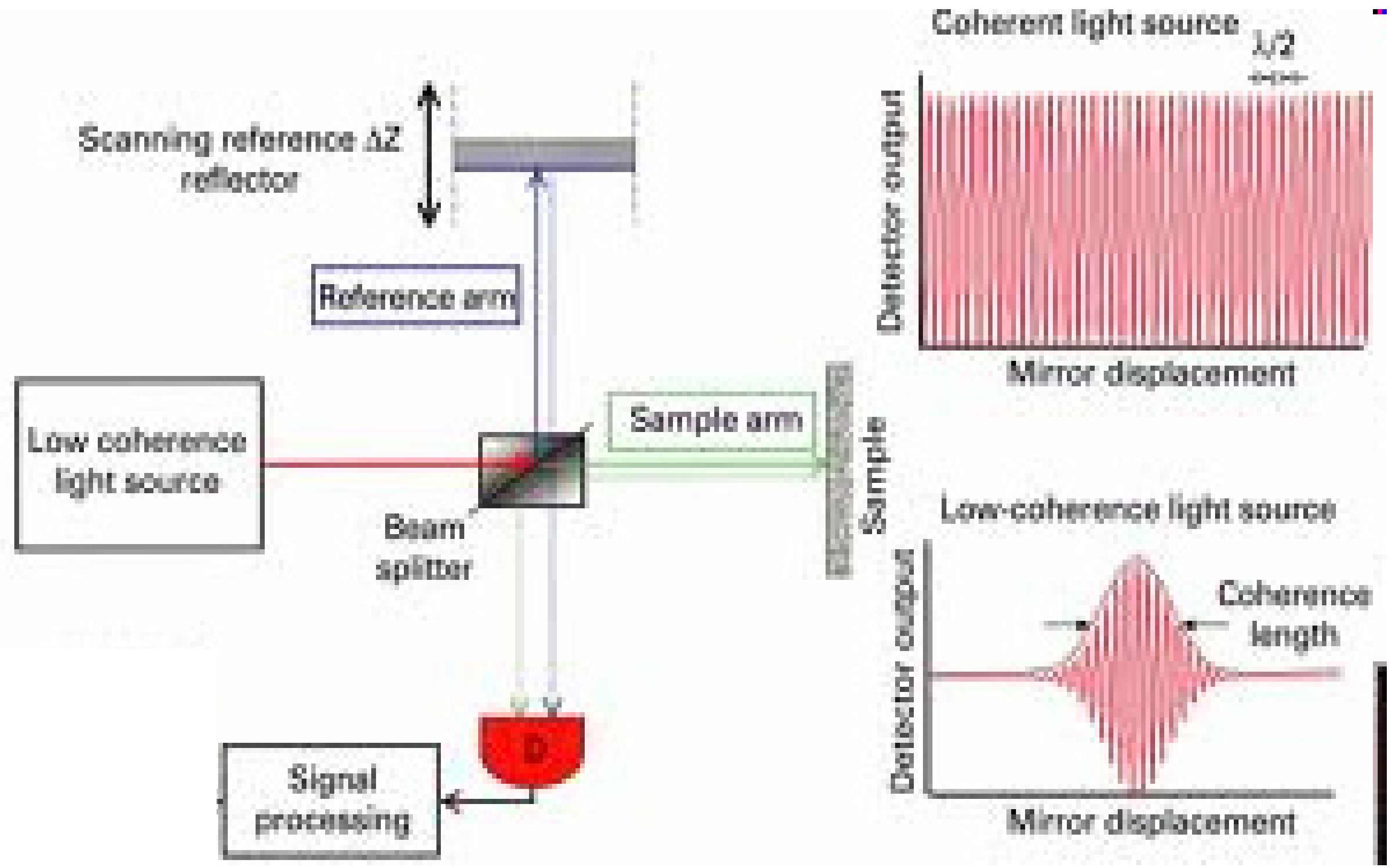
## Temporal coherence:

coherence length = c* coherence time

## Spatial coherence:

coherence area

# Measuring coherence length by interferometer



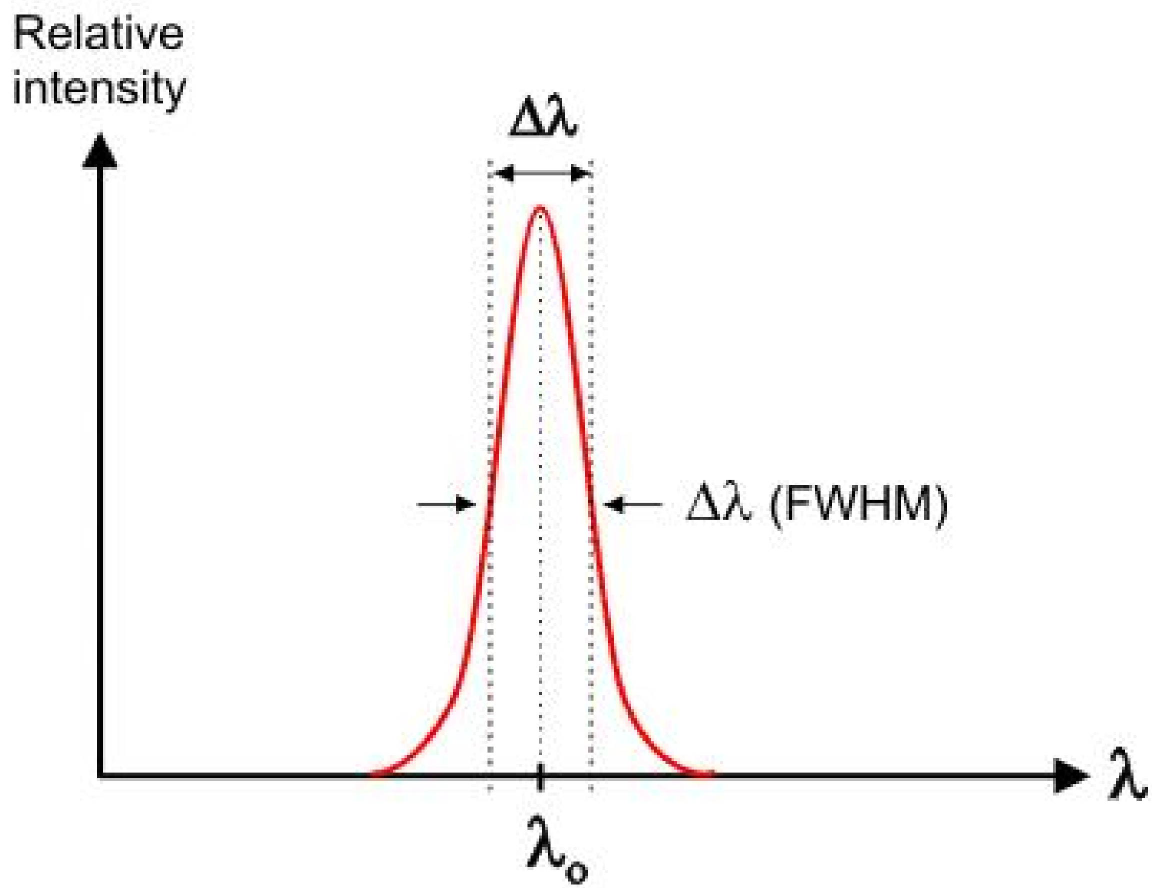
# Coherence lengths of light sources

| Source | Mean Wavelength $\bar{\lambda}$(nm) | Linewidth* $\Delta\lambda$ (nm) | Coherence Length $\Delta l_c$ |
|---|---|---|---|
| Thermal IR (8000–12 000 nm) | 10 000 | $\approx$4000 | $\approx$25 000 nm = $2.5\bar{\lambda}$ |
| Mid-IR (3000–5000 nm) | 4000 | $\approx$2000 | $\approx$8000 nm = $2\bar{\lambda}$ |
| White light | 550 | $\approx$300 | $\approx$900 nm = $1.6\bar{\lambda}$ |
| Mercury arc | 546.1 | $\approx$1.0 | $\leqslant$ 0.03 cm |
| $Kr^{86}$ discharge lamp | 605.6 | $1.2 \times 10^{-3}$ | 0.3 m |
| Stabilized He-Ne laser | 632.8 | $\approx 10^{-6}$ | $\leqslant$ 400 m |
| Special He-Ne laser | 1153 | $8.9 \times 10^{-11}$ | $15\times10^{6}$ m |

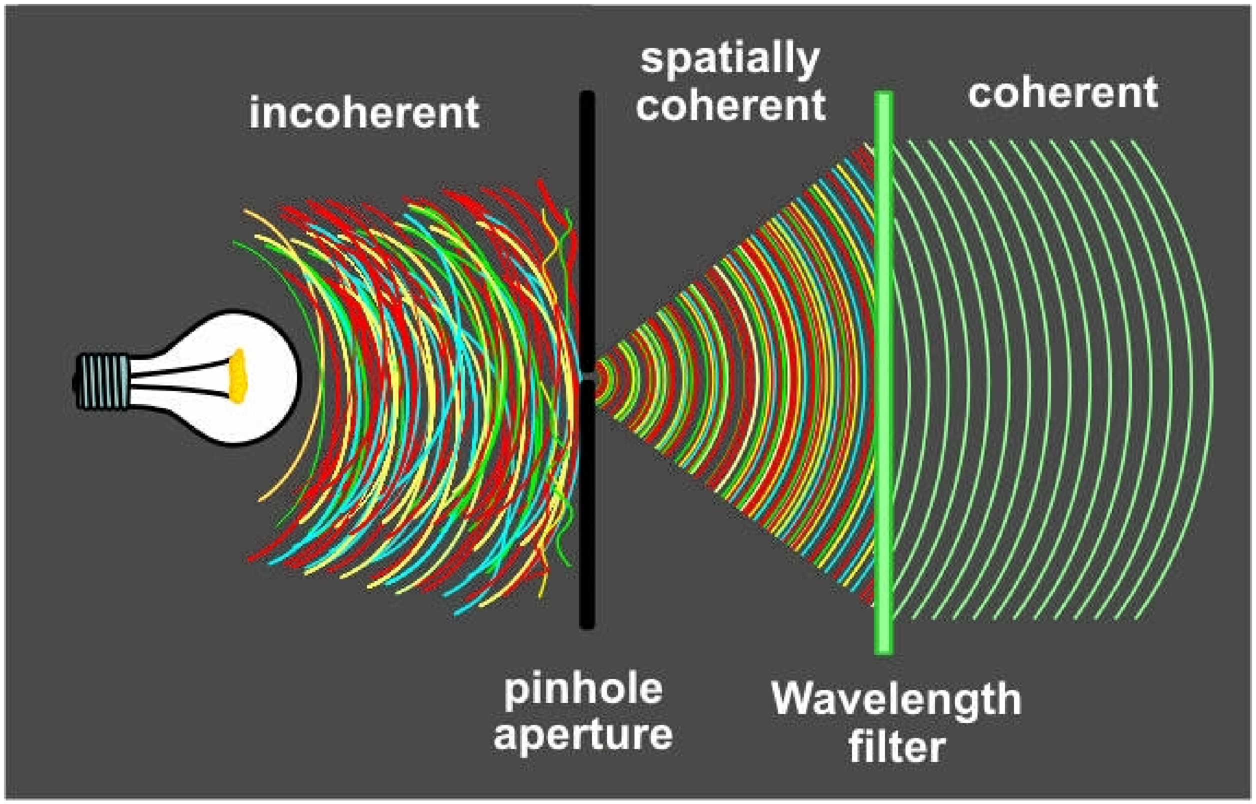
# Coherence length and linewidth

Narrow linewidth → long coherence length

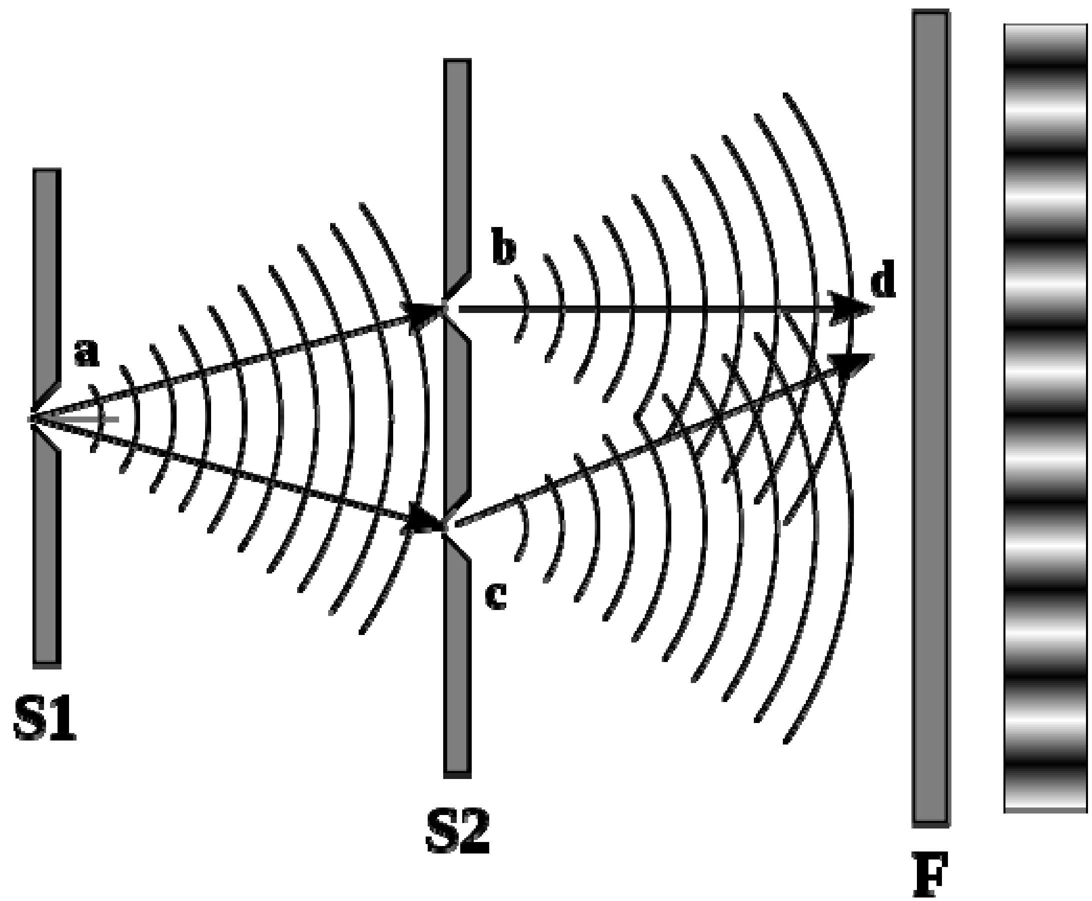
Wide linewidth → short coherence length



# How to improve degree of coherence ?

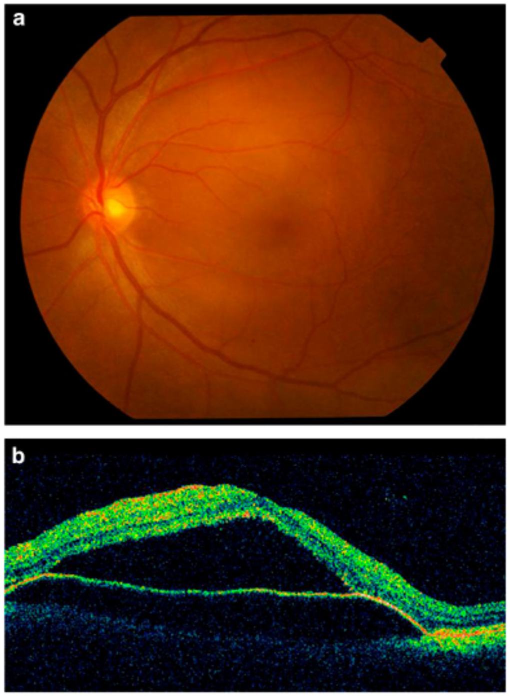


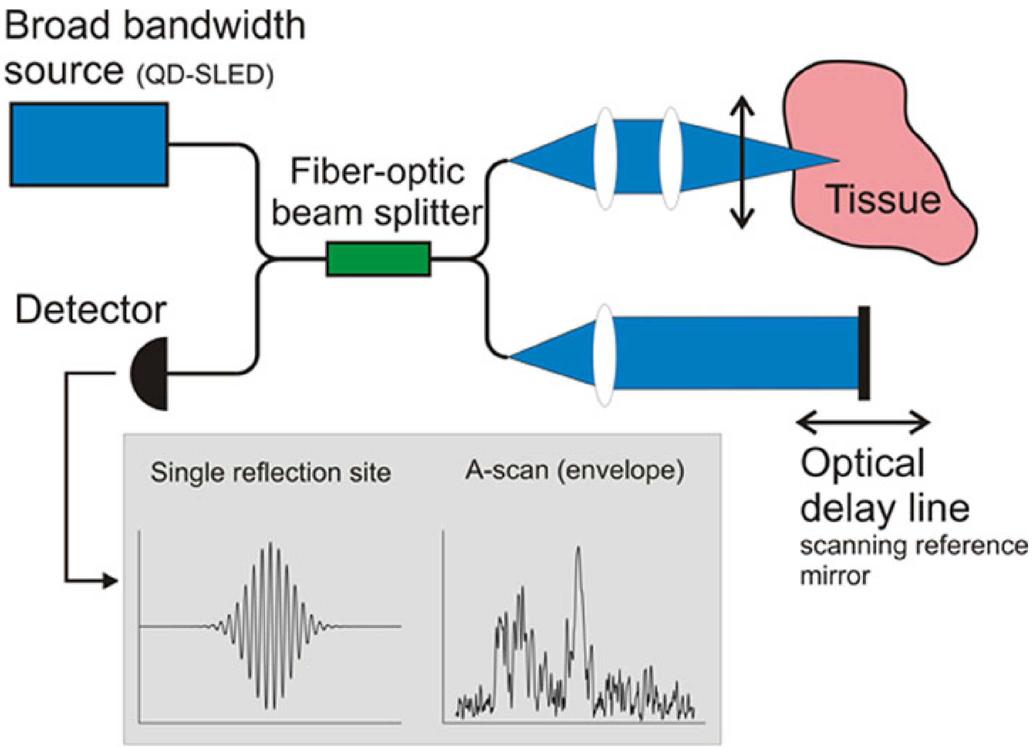
# Spatial coherence in interference



# Optical coherence tomography

- Broadband source
- Depth resolution is given by low coherence interferometry



# Photon theory of light

A *photon* is a discrete bundle (or *quantum*) of electromagnetic energy.

• move at a constant velocity, $c = 2.9979 \times 10^8$ m/s (i.e. "the speed of light"), in free space

• have zero mass.

• carry energy and momentum, which are also related to the frequency $\nu$ and wavelength $\lambda$ of the electromagnetic wave by

• $E = h\nu$ **and** $p = h/\lambda$.

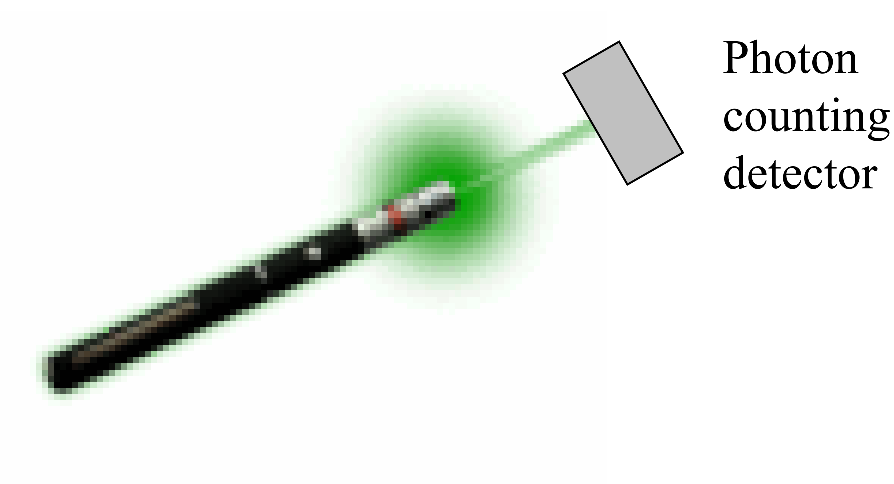
• can be destroyed/created when radiation is absorbed/emitted.

• can have particle-like interactions (i.e. collisions) with electrons and other

# Photons carry energy

Exercise:

How big is the photon flux in a 1mW green laser beam?

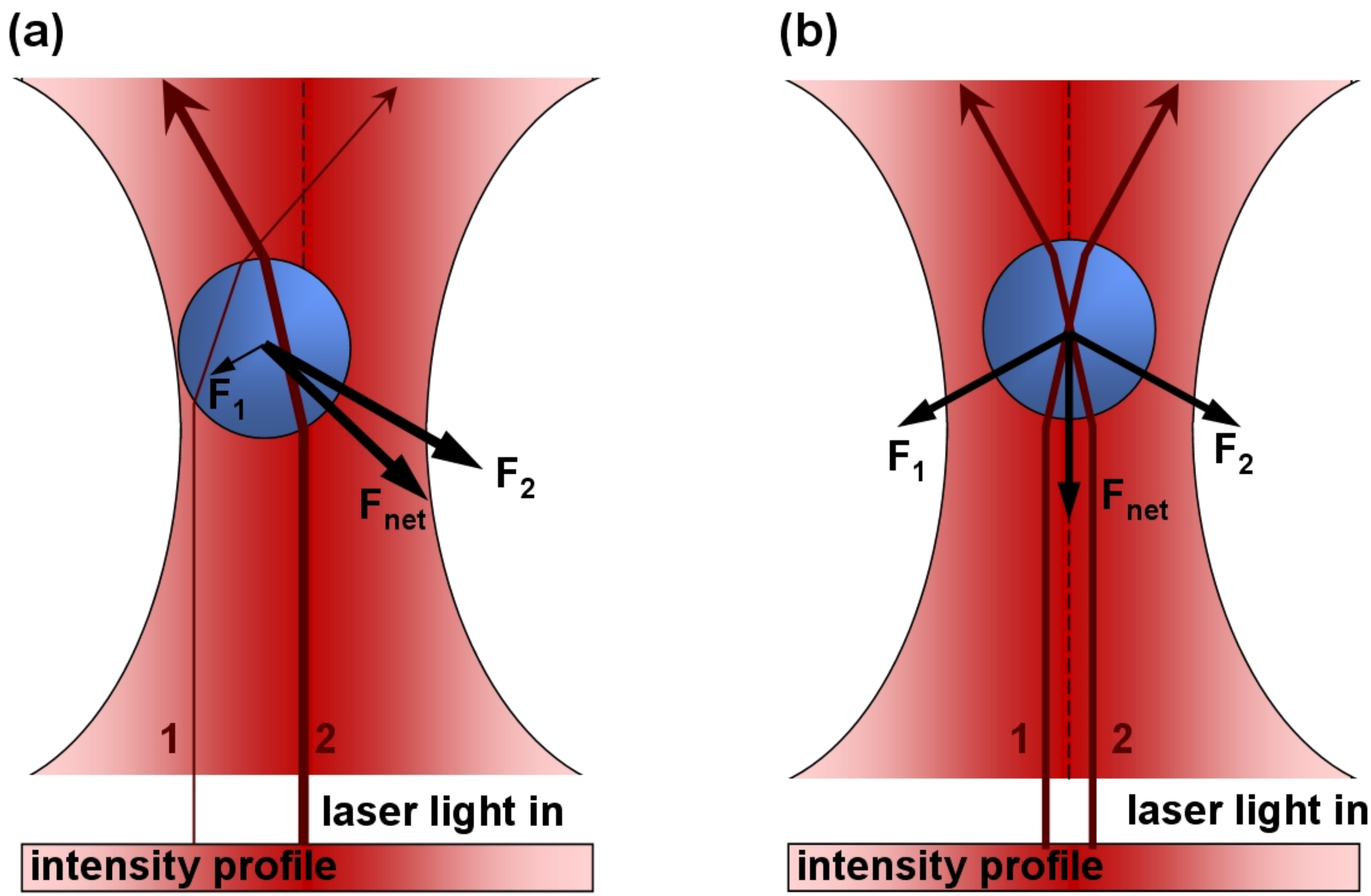



$$E = h\nu.$$

$$E = \frac{hc}{\lambda}.$$

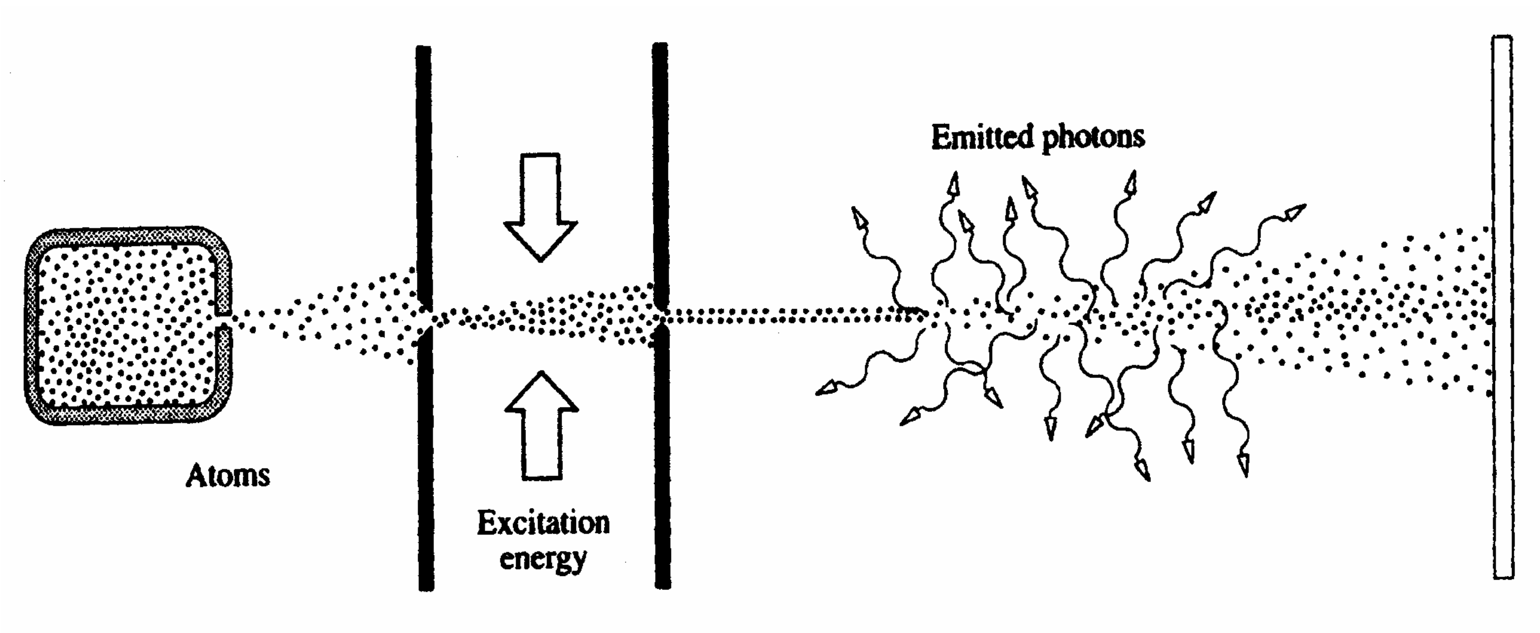
**Planck's constant = 6.6 × $10^{-34}$ J * s**

# Optical tweezer



# Photons carry momentum

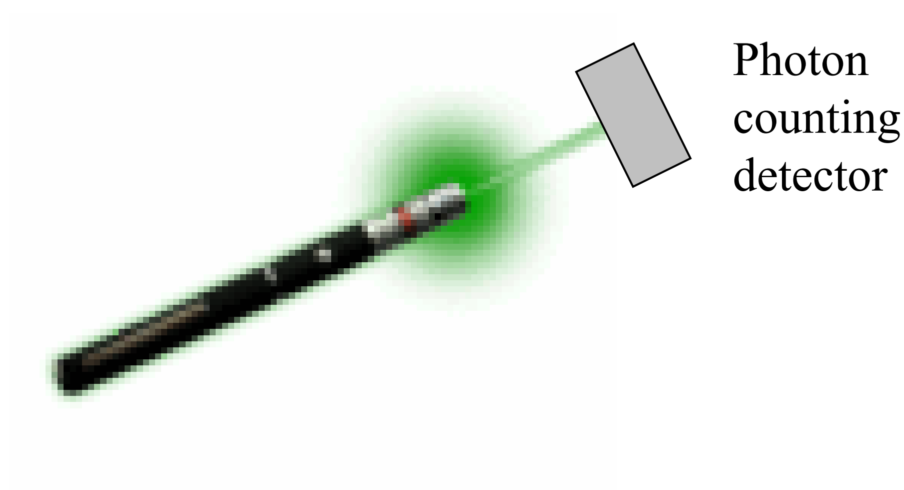
If an atom emits a photon, it recoils in the opposite direction.



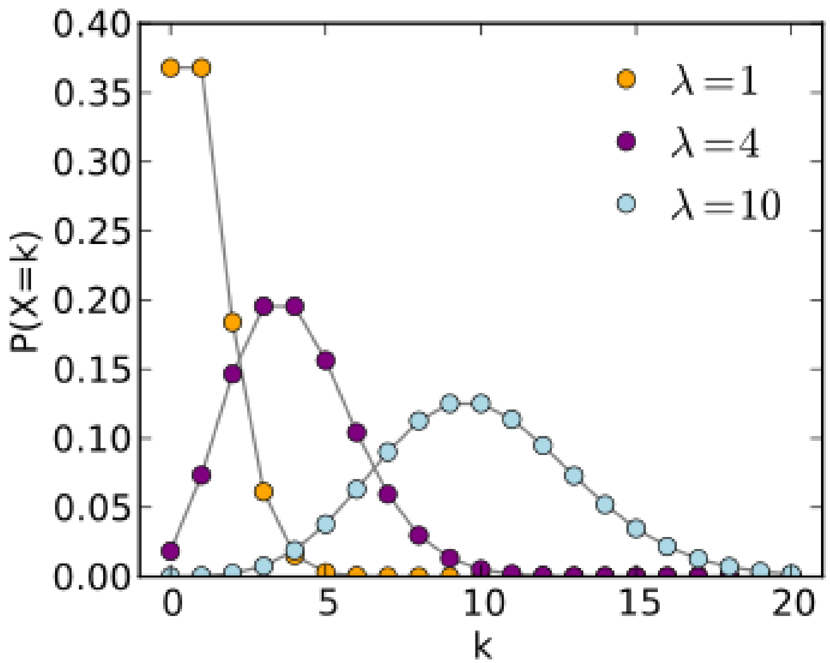

If the atoms are excited and then emit light, the atomic beam spreads much more than if the atoms are not excited and do not emit.

# Photon counting

What is the actual number of photons if we detect a 1mW green laser beam within 1 picosec ?



# Poisson distribution




$$f(k;\lambda) = \frac{\lambda^k e^{-\lambda}}{k!},$$

The parameter λ is the mean of the distribution;

k is the variable number of events occurring.

A discrete probability distribution that expresses the probability of a certain number of events occurring in a fixed period of time, if these events occur with a known average rate and independently of the time since the last event.

# Generality of Poisson distribution

Poisson distribution applies to various phenomena of discrete properties (that is, those that may happen 0, 1, 2, 3, ... times during a given period of time or in a given area) whenever the probability of the phenomenon happening is constant in time or space, and the events are independent of each other.

- The number of photons hitting a photodetector per unit time.

- The number of phone calls at a call center per minute.

- The number of raindrops falling over an area per second.

- The number of mutations in a given stretch of DNA after a certain amount of radiation

# Shot noise, quantum noise, Poisson noise

$$f(k;\lambda)=\frac{\lambda^k e^{-\lambda}}{k!},$$

Variance = Mean (problem set)

Standard deviation = Square root of mean

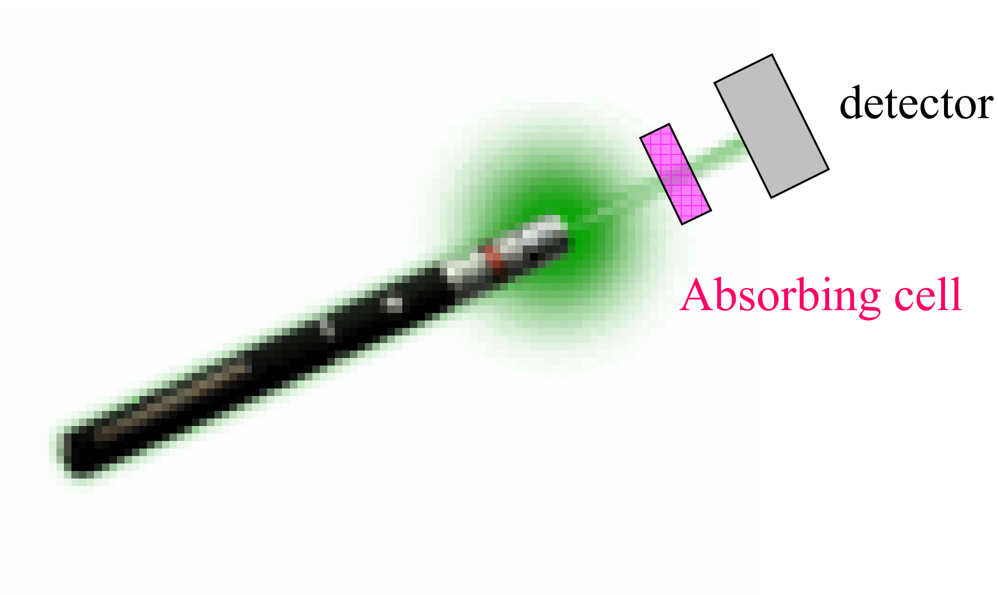
$$\mathrm{SNR}=\frac{N}{\sqrt{N}}=\sqrt{N}$$

Magnitude of this noise increases with the average magnitude of the current or intensity of the light.

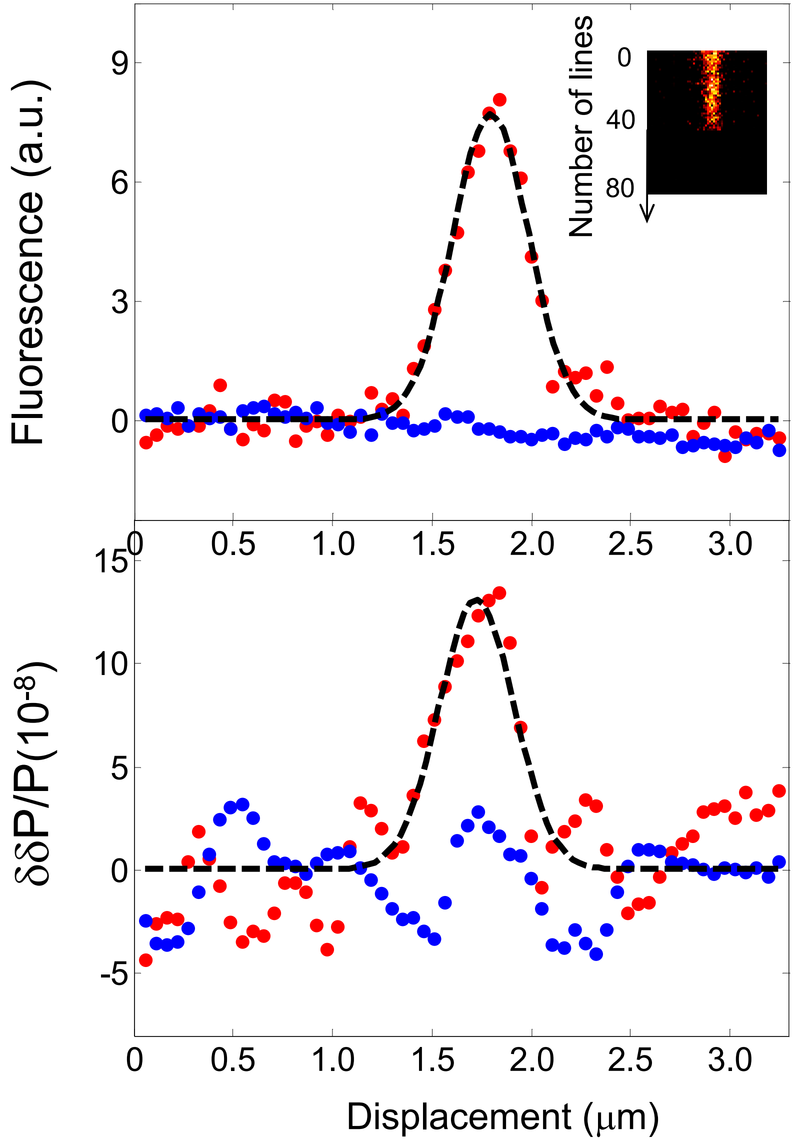
However, since the magnitude of the average signal increases more rapidly than that of the shot noise, shot noise is often only a problem with small light intensities.

# Shot-noise-limited absorption detection

How sensitive can we detect the absorption signal with a 1mW green laser beam within 1 picosec?

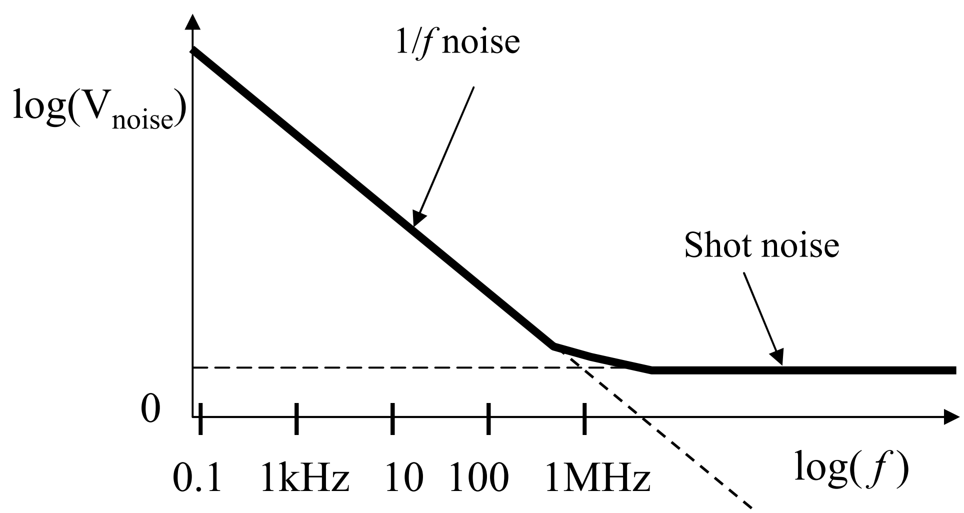


# Shot-noise-limited detection sensitivity



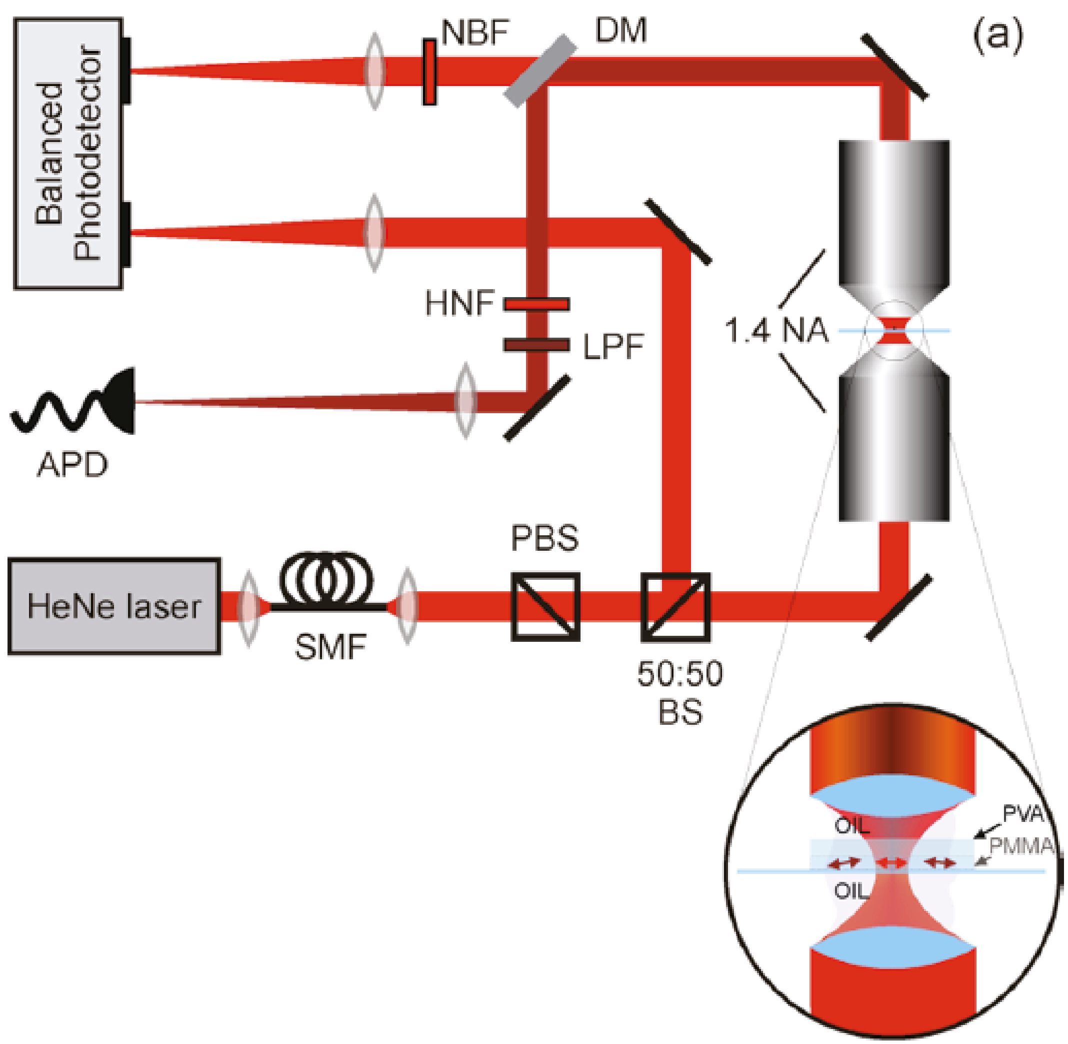

Detection of absorption signal by a single molecule

**Noise spectrum in frequency domain**

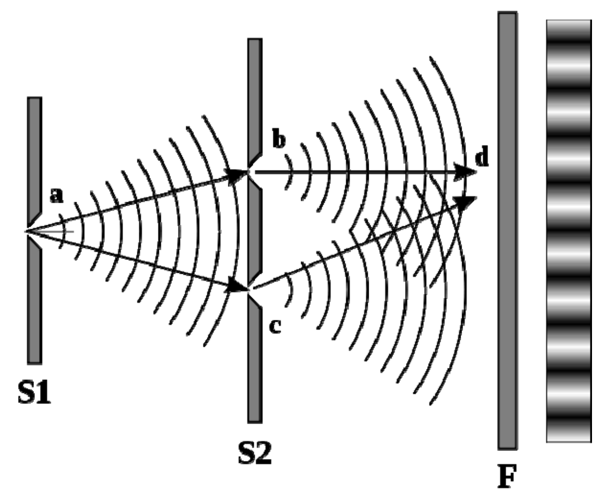


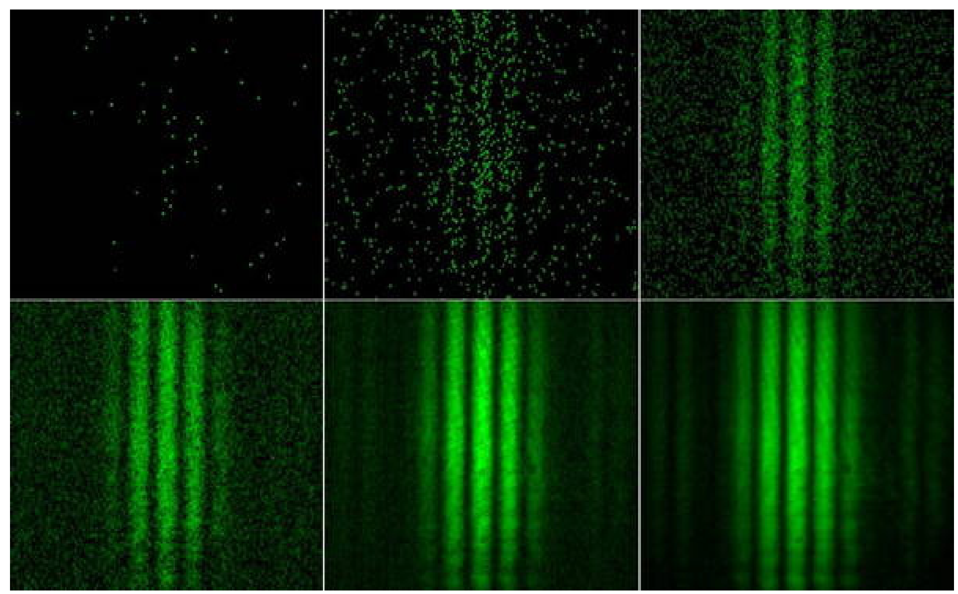

Chong, Min, Xie. *JPCL, 2010*

# Common mode rejection

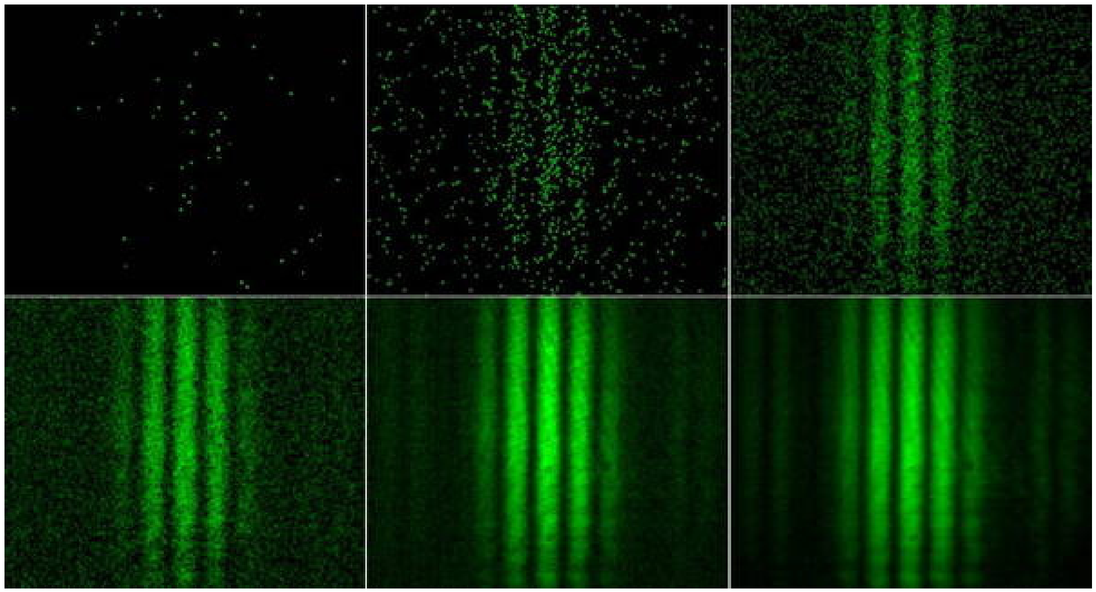


# Wave-particle duality of light



# Probability wave



Probability wave function represents the probability of finding a given particle at a given point.

These probability waves can diffract, interfere, and exhibit other wave-like properties.

# Single photon interference

“ “Some time before the discovery of quantum mechanics people realized that the connection between light waves and photons must be of a statistical character. What they did not clearly realize, however, was that the wave function gives information about the probability of **one** photon being in a particular place and not the probable number of photons in that place. The importance of the distinction can be made clear in the following way. Suppose we have a beam of light consisting of a large number of photons split up into two components of equal intensity. On the assumption that the beam is connected with the probable number of photons in it, we should have half the total number going into each component. If the two components are now made to interfere, we should require a photon in one component to be able to interfere with one in the other. Sometimes these two photons would have to annihilate one another and other times they would have to produce four photons. This would contradict the conservation of energy. The new theory, which connects the wave function with probabilities for one photon gets over the difficulty by making each photon go partly into each of the two components. Each photon then interferes only with itself. Interference between two different photons never occurs.” ”

**—Paul Dirac, The Principles of Quantum Mechanics, Fourth Edition, Chapter 1**

# Classical wave picture vs. quantum mechanics

Electric field $E_i$ (with phase information)

↓

Interfere with each other to produce the final E

↓

Detectable light intensity
$I \sim E^2$

Probability amplitude (with phase information)

↓

Sum over all probability amplitude of all possible paths to produce the final probability amplitude

↓

Observable probability of certain event: squared modulus of the final probability amplitude

# Trigonometry vs. complex number

Classic wave picture

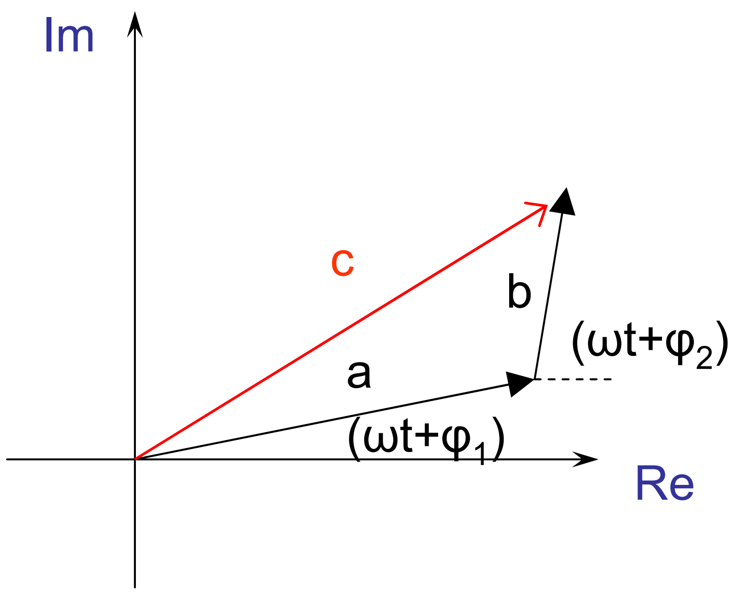
$$E_1+E_2 = a*\cos(\omega t+\varphi_1) + b*\cos(\omega t+\varphi_2)$$

Complex number addition

$$E_1+E_2 = a*\cos(\omega t+\varphi_1) + b*\cos(\omega t+\varphi_2)$$

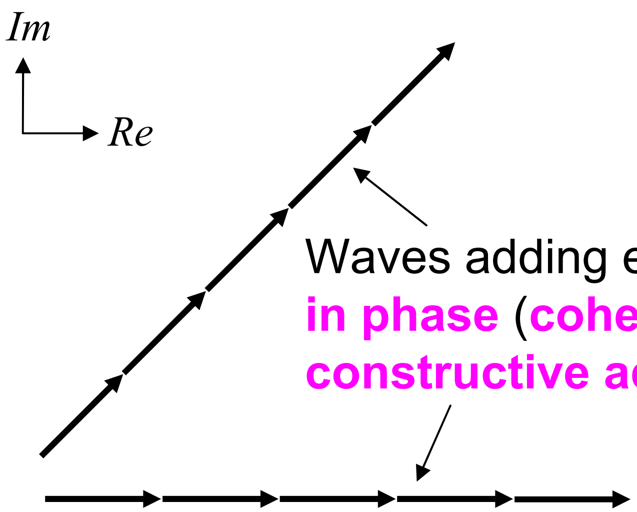
$$= a* \mathrm{Re}\{\exp[i(\omega t+\varphi 1)]\} + b* \mathrm{Re}\{\exp[i(\omega t+\varphi 2)]\}$$

$$= c* \mathrm{Re}\{\exp[i(\omega t+\varphi 3)]\}$$

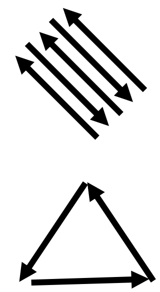



$$e^{i\theta} = \cos\theta + i\sin\theta.$$

# Scenarios using complex numbers




Waves adding exactly **in phase** (**coherent constructive addition**)



Waves adding exactly **out of phase**, adding to zero (**coherent destructive addition**)

Waves adding with **random phase**, partially canceling