Conductivity Lab Overview

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1 How to Proceed

Welcome to the solid state electrical conductivity lab! This set of experiments introduces you to concepts and procedures that are essential to the modern understanding of the electrical conductivity of solids.

This overview sketches out the main ideas and structure of this lab. All of the gritty, fun details are placed in various **Notes**; the theory is expanded in various **References**, but an overview of the essential experimental techniques is presented in the very next section.

The first thing you should do is read the rest of this document, with an eye to laying out an experimental program. You will find that the first thing you'll actually have to do is to calibrate of a Pt sample and a commercial diode for low temperature thermometry; but after that, you can measure whatever you want, subject to availability.

If you are going to be making your own superconducting samples, you should then read the notes on **Making BSCCO**. That process takes a long time, most of which is spent waiting for the BSCCO grains to grow. You should start that process immediately, so that the growth can occur while you learn how to use the apparatus and conduct your preliminary measurements.

2 Techniques You Will Be Using

2.1 The Alternating 4-Wire Technique

A voltmeter doesn't know what it's connected to. If you were to measure the voltage across the sample using the same leads with which you applied the probing current, you would also measure the voltage drop through the current leads. Since the leads are long and the sample resistance may be small, this can be a very significant source of measurement error.

The simplest solution to this problem is to apply the current with one pair of wires, and measure the voltage with a second pair.

Figure 1: Four-Wire Schematic.

Another source of extraneous voltage is the *Seebeck effect* due to the thermal gradient down the voltage leads. Warm electrons diffuse faster down the wire, leading to a voltage proportional to the temperature difference. This material-dependent effect is the basis for the thermocouple thermometer, but here it's a nuisance because inevitable asymmetries in the lead wires will lead to a voltage offset. The solution to this problem is to alternate the direction of the applied current. In the average of the two measured voltages, this and other asymmetries cancel out.

The Keithley current source and voltmeter are set up for this measurement in what Keithley calls "Delta mode" (more information is in the **Keithley notes**). You can initiate these measurements either from the Keithley front panels, or from the scripts we have set up for you in Matlab (see the **Matlab Script Notes**). If you are interested, you may also apply the Keithley in DC mode to see the magnitude of the lead asymmetry.

2.2 Refrigeration

To get samples down to low temperatures, you have the use of a closed-cycle helium refrigerator. Compared to the cryostats used in the other low temperature labs, this system is more like a refrigerator in the sense of a kitchen refrigerator, with a compression/expansion cycle of a refrigerant fluid. The important details of how to operate this rather complicated bit of machinery are discussed in the **CTI Refrigerator Manual**. However, if everything is working properly, the instructions on **Cooling Down** should suffice.

For now, note that the refrigerator comes in two parts, connected by pressurized helium hoses. The big box contains a large pre-compressor; the heavy object to which these leads go is called the *cold head*. It is so clunky because it contains a motor that drives heat-exchanging fluid displacers back and forth. Once it's been wired up, the sample to be measured is thermally anchored to the coldest part of the cold head–the tip of the *cold finger*. The thermometer that you calibrated put into good thermal contact with the sample. Note that all of the lead wires are thermally anchored all the way down the cold finger to prevent unmitigated thermal conduction from the feedthrough plugs to the sample.

Next, two heat shields are placed around the sample. These fairly isothermal surfaces both slow radiative heating from the tip of the cold finger and ensure that the environment around the sample is at a constant temperature. Finally comes the outer tail, which pressure-seals around the shielded cold finger. Once this is done, a vane pump gets most of the atmosphere out of the sealed-off space (otherwise the remnant vapor pressure of the eventually frozen air would leak heat to the sample). At this point, with everything canned up and evacuated, you are ready to start the refrigerator.

Unburdened with a heat load, the refrigerator is meant to cool the tip of the cold finger down to around 10K over about 60 minutes. Once your sample has reached base temperature, you can heat it by using the heater control on the Lakeshore thermometer box. This puts current through a stretch of NiCr wire in thermal contact with the sample holder. The Lakeshore box uses a Proportional-Integral-Derivative (PID) feedback algorithm, driving the thermal mass of the thermometer like a damped harmonic oscillator to establish the appropriate current through the heater coil.

Figure 2: Rig Block Diagram.

2.3 Platinum Thermometry

Well and good: but how to establish what the temperature is? What's required is a well-characterized process that overlaps a trusted temperature scale at the high end, but persists to lower temperatures. Over a wide range of temperatures (as measured by, say, gas bulb thermometers filled with helium, our best real approximation of a classical ideal gas), platinum has a very linear resistivity. By calibrating a sample of platinum, you should be able to extend your practically measurable temperatures downward to about 30 K.

Figure 3: Pt Resistivity.

A temperature standard provides a continuous function of some physical quantity versus temperature. The usual choice for Pt resistors is the phenomenological *Callendar-Van Dusen function* (really two functions spliced together), about which you can read in the **Pt Calibration References**:

$$R(T) = R_0(1 + AT + BT^2 + C(T - 100)T^3).$$

Here the term with C is applied only for $T < 0^\circ$ centigrade. This equation has four parameters, which you can determine for your particular bit of wire by comparing the measured resistances at *fixed points*, determined by phase transitions.

Two obvious fixed points are the temperature of water in equilibrium with ice, and the temperature of water in equilibrium with vapor, at a well-defined pressure. But since thermometers based on this scale are common, you can in fact establish the first three Callendar-VanDusen constants by measuring the resistance of your Pt in equilibrium with a mercury expansion thermometer at at least three different temperatures. You will have to decide how precise the mercury thermomter values are, and how these uncertainties propagate into your Pt calibration. Is your calibration for $T > 0$ C self-consistent within these uncertainties? How well does your calibration pin down the melting point of Gallium (about 302 K)?

The last constant requires a measurement at low temperature. A convenient fixed point here is the temperature of boiling liquid nitrogen at (approximately) one atmosphere, which is about 77 K. You will have to decide whether variations in the ambient pressure warrant a correction to the nominal value. You may check the robustness of your Pt standard against other fixed points from the **Data References**.

To use your platinum in the refrigerator, you have to figure out how to overcome some difficulties associated with measurement at low temperature. You want to make sure that there is good thermal contact (and poor electrical contact) between your samples and the putatively cold sample stage, and you want to make sure that your sample doesn't become mechanically strained as it cools and contracts, as this could lead to extraneous hysteresis. For more details, see the **Sample Mounting Notes**.

2.4 Semiconductor Thermometry

Below 30 K, the resistivity of a platinum sample becomes noticeably less linear. This is the crossing from phonon-dominated scattering to temperature-independent impurity scattering. On the other hand, the resitivity of a semiconductor increases as thermally excited carriers freeze out. Hence, just where platinum thermometers become useless, the appropriate semiconductor thermometer will have an increasing sensitivity. By modeling the conductivity of a as a function of temperature, and calibrating this against a platinum resistor, you may again push down the lower bound of your temperature sensitivity. However, you'll need a different function to describe the resistivity.

This is the basic idea behind the silicon diode thermometer. Since diodes are rather more artificial than pure platinum, the particulars of the diode resistivity curve depend on the details of its manufacture. Luckily, the Lakeshore controller has an automatic algorithm for fitting to a Lakeshore silicon diode, so it is pretty easy to calibrate this one, now using your Pt resistor as a new standard. For details on this "SoftCal" procedure, check the **Lakeshore Manual**.

3 Electrical Conductivity

The electrical conductivity (σ) defines the response of a system to an applied electric field. The conductivity measures the tendency of electrical charges to move through the solid because of the field. This of course is the empirical property that makes metals metallic and insulators insulate, depending on the values that it takes. I can't overstate its technological significance.

In practice, one extracts information about the DC electrical conductivity from the resistivity (ρ), which is the inverse of the electrical conductivity ($\rho = 1/\sigma$). For the purposes of this lab, you can begin thinking of the resistivity as a shape-independent, bulk property belonging to a kind of material. The approximate separation of bulk properties from geometric specifics is what allows people to make differently rated resistors from the same roll of wire.

Figure 4: Resistivity Cartoon.

3.1 The Drude Model

The standard classical mechanical model for the conductivity/resistivity of solid materials is called the Drude model. In the DC version of this model, which should be familiar from your introductory physics courses, the electron is treated like a little charged ball, bouncing along through the material under the influence of an accelerating electric field. After every bounce, the velocity is reset to some random value consistent with the temperature of the lattice. Unless there is something to stop the electrons–some sort of boundary or tether–they will tend to rattle on their way forever, with a bias in the direction of the electric field. So in the absence of an explicit binding force, this DC model describes a metallic conductor. The presence of a binding force implies electrical insulating behavior.

For AC voltages, the distinction between conductors and insulators is muddied, because AC currents in metals go back and forth anyway. Nevertheless, one expects a qualitative difference based on the notion that bound, oscillating electons should have a resonance frequency (corresponding to the“spring constant” of their tether). So one can generally define a metal in the Drude model as a material for which at least one “resonance” occurs at zero frequency: at least one “spring constant” is set to zero.

Figure 5: The Drude Picture.

It's pretty simple to derive the full AC-DC Drude model for conductivity. To start, assume that a single charge carrier (mass m and charge q) is accelerated under the influence of an electric field with a form $E(t) = E_0 \cos \omega t$, while it is also subject to a restoring force kx (x is the displacement from equilibrium) and a dissipative force fv (v is the velocity). Solve the Newtonian equation to find the amplitude and maximal speed of the steady-state solution. One slick way to do this is to imagine the complex field $E(t) = E_0 e^{i\omega t}$ and take the real part at the end of the calculation. This works because the differential equation is linear. Next note that the classical expression for the electrical current density j is just $j = nq < v >$, where n is the number density of the charges, and the brackets indicate a sample average. Plugging in your expression for v, you'll get

$$\textsc{j}(t) = \frac{nq^2 \times i\omega E_0}{k - m\omega^2 + i < f > \omega} \times e^{i\omega t}.$$

The proportionality between j and E is the frequency-dependent conductivity ($\sigma(\omega)$ for the frequency ω). If you rationalize the denominator and take the real part of your answer, you will find that it falls into two pieces: one part of the current is in phase with the electric field (goes like $\cos(\omega t)$) and the other is $\pi/2$ out of phase (goes like $\sin(\omega t)$). What does this mean?

jE has the units of a power density. Over a cycle, the out-of-phase product $\sin\cos$ averages to zero, but the in-phase $\cos^2$ averages to $\frac{1}{2}$. This nonzero part is the cycle-averaged power transmitted to the current. Where does it go? Its proportionality to $< f >$ is an important hint. Recall that f represented a damping force on a single oscillating charge. What we have described here is a steady state: the kinetic energy of the current is dissipated by the frictional force. By contrast, the out-of-phase part determines the energy flow into and out of the electromagnetic fields with no net absorption.

Because we initially assumed $k \neq 0$, we have described a DC insulator. However the same analysis goes through with $k = 0$, so the expression for the DC conductivity for conductors is just the $k = 0$ limit of the general expression. This leads to another familiar interpretation of the averaged dissipation coefficient $< f >$. Given that the additional velocity of a charge accelerated in a DC field E for time τ should be $qE\tau/m$, the current density should be $nq^2E\tau/m$. Comparing the two expressions for the DC current density shows that $< f >= m/\tau$. In other words, $m/ < f >$ is the average time for the uniform acceleration of an electron by the external field, before the electron hits some obstacle and decays out of the current.

3.2 "Perfect Conductors"

Things get a bit subtle when we try to imagine what would happen for a perfect DC conductor ($k = 0$ and $< f >= 0$ simultaneously). Imagine a sequence of graphs of the in-phase part of the conductivity as $\tau \to \infty$ and $< f > \to 0$. The typical Lorentzian narrows, and the peak at $\omega = 0$ simultaneously increases, in such a way that the total area under the curve remains constant. If we rescale the ordinate, we get closer and closer to what seems a perfectly thin spike at $\omega = 0$. This $< f > \to 0$ limit is a Dirac δ-function. For a perfect conductor, with no dissipation, the in-phase conductivity happens at zero frequency. Because τ represents the amount of time for which an electron retains the energy that the field gives it, and (for constant m) $< f > \to 0$ makes $\tau \to \infty$, a perfect conductor would be a great place to store kinetic energy, transmit energy without loss, etc. If only life were that easy!

Figure 6: Narrowing Lorentzians.

3.3 Temperature dependences of the Scattering

3.4 Impurity Scattering in Metals

The idea of perfect metallic conductivity is no relic of the Drude model. The quantum mechanical treatment of the electron states in a lattice also implies that a single electron need not scatter out of its eigenstate. So why do metals exhibit finite τs? There are many mechanisms to consider. The simplest to imagine is impurity scattering. An electric field gives a directed momentum to a conduction electron; the scattering centers mix up momenta. Hence if impurity scattering were the only scattering process, one would expect a mean free path of about the average impurity spacing, and a corresponding *impurity scattering resistivity* that could be made as low as one wished by careful materials processing.

To be specific, consider the following ballistic model for impurity scattering. Call the density of impurities is N_i. Define the *mean free path*, L, as the length for which a charge carrier on average hits one impurity. Actually, it's easier to imagine the impurities sweeping toward the charge carrier. Supposing that the radius of one impurity is r_i, then $L \times \pi r_i^2 N_i \sim 1$. But $L = v\tau$, where v now stands for the average speed. So $\tau \sim \frac{1}{vN_i}$. All else being constant, $N_i \to 0$ implies $\tau \to \infty$.

For a fixed N_i, we might expect to be able to understand temperature dependence from the variation of v. If, as you might expect for a classical gas, v were something like $\sqrt{k_B T/m}$ (with $k_B T >> E\tau$) then you might expect the impurity resistance to increase with temperature.

In fact the proper speed to use at all temperatures for which the metal is solid is the Fermi speed, v_F, which for classical particles would correspond to a temperature on the order of 10^4 to 10^5 K. The fact that in metals the impurity resistivity is largely independent of temperature is a consequence of the fact that electrons do not obey classical statistics.

This consideration leads to one idea for an experimental program. If $\tau \sim \frac{1}{v_F N_i}$, then you can estimate v_F of a metal by introducing a controlled number of impurities. If you are interested in pursuing the matter, then as a first step, see if you can work out the details of how you would extract v_F from a data set of ρ versus N_i.

3.5 Carrier Freezeout in Semiconductors

In nonmetals, the freeing of a charge requires an activation energy. As the temperature increases, one expects a population of thermally excited carriers. Recall that the Drude-model conductivity $\sigma = \frac{nq^2\tau}{m}$. With thermal excitation, $n \propto e^{-\Delta/k_B T}$, where Δ is the excitation energy. Hence (ignoring for the moment the much weaker temperature dependence of τ) you'd expect the resistivity of a semiconductor to rise dramatically at lower temperatures.

So here we have another possible experiment: the exponential temperature dependence of the conductivity might allow one to extract information about the *band gap* energy Δ. If you are interested in pursuing this matter, then as a first step, see if you can imagine how to extract Δ from a data set of ρ versus T.

For semiconductors, the calculation of the scattering time is a bit more complicated that it is for conductors. Here is one way to grasp the main result. The *thermal wavelength* of a quantum mechanical particle of mass m at temperature T is

$$\lambda_T = \sqrt{\frac{\hbar^2}{2mk_BT}}.$$

This defines a thermal volume V_T per particle of $\lambda_T^3 \sim (k_BT)^{-2/3}$. Now N_iV_T will be the number of scatterers in the same volume, so you would expect that for the impurity scattering rate, $1/\tau \sim (k_BT)^{-2/3}$. As promised, the temperature dependence of τ in semiconductors is much weaker than that of n. The reason the same analysis fails in metals is that for metals, the appropriate temperature to use for the wavelength of a particle is the Fermi temperature T_F (with $T_F >> T$), which is proportional to $n^{2/3}$. In metals the average spread of the charge carrier wavepackets is much smaller than the typical impurity spacing–and so there the ballistic approach works better.

3.6 Phonon Scattering in Metals

There are other effects to see. Even a perfect atomic lattice will support normal modes, known as *phonons*, which cause local variations in the density that act as scattering centers. So the average density–of these density variations–should also limit the mean free path and contribute to the resistivity. Electrons will scatter from other electrons. As long as there is something from which to scatter, and an open state for the electron to go into (recall that electrons are Fermi particles) our Drude-model currents, once set up, will eventually decay.

Faced with the multiplicity of scattering mechanisms, you might not be surprised that the general computation of the electrical resistivity of a metal can turn into a very complicated undertaking. The total scattering rate should be something like the sum of all the different rates taken individually. That is, $\frac{1}{\tau_{total}} = \frac{1}{\tau_{impurity}} + \frac{1}{\tau_{phonon}} + \ldots$. In other words, all else being constant, the various resistivities of the material should add like resistors in series. There is some hope of disentangling the major contributions because each mechanism has a characteristic temperature dependence.

Impurity scattering should dominate at the very lowest temperatures where there is not

Figure 7: Phonon Cartoon.

enough thermal energy around to excite many of the excitations which cause other scattering processes. Phonons of a given wavelength require a characteristic quantum of energy, which is larger for the phonons of larger wavevector that would be most effective in turning charge carriers around. Below the *Debye temperature*, Θ_D, the number density of high-wavevector lattice vibrations decreases dramatically with temperature. So below Θ_D, one expects the effect of phonon scattering to lessen and the resistivity to decrease. Well above Θ_D, the temperature dependence of the resistivity results from a combination of a gentler temperature dependence of the number of phonons. Here's the classic (but not classical) formula (and don't let it scare you):

$$\frac{1}{\tau} = v_F \sigma_a \frac{\hbar^2 q_D^2 k_B}{M_a k_B^2 \Theta_D} \left(\frac{T}{\Theta_D}\right)^5 \int_0^{\Theta_D/T} \frac{4z^5 dz}{(e^z - 1)(1 - e^{-z})}.$$

This, the *Bloch-Gruneisen formula* for the phonon part of the resistivity in metals predicts that for $T << \Theta_D$, $\rho_{phonon}(T) \sim (\frac{T}{\Theta_D})^5$, while in the opposite limit, the temperature dependence is linear. It's not a perfect formula, because it ignores the enhanced effect of long-wavelength phonons known as *umklapp scattering* as well as interband and electron-electron scattering processes. But it's qualitatively correct for many metals, especially the alkalis, where the smallest wavevector for umklapp scattering is comparable to the Debye wavevector q_D. For more on Bloch-Gruneisen and its brethren, see the **Conductivity Theory References**, especially the **Tabulation of the BG Transport Integrals**, which is useful for actually fitting data.

So considering both phonon and impurity scattering together, you would expect the DC resistivity of a metal to increase as a function of temperature, and at low temperature decreases to a finite value. In terms of the Drude parameters, for metals, n is a constant. Likewise $\tau_{impurity}$ is constant, but τ_{phonon} increases. By fitting the ρ versus T data for a metal, you can estimate the experimental quantities, Θ_D, n, $\tau_{impurity}$, and the jumble of other constants that determines the overall scale for τ_{phonon} You can see the **Conductivity Theory References** for some examples of this kind of analysis.

3.7 Conductivity Measurement Summary

So the big idea is to mount a sample on the tip of the cold finger, change the temperature, and measure the resistance of the sample as a function of temperature. For different kinds of samples, you should see qualitatively different behaviors.

Below are a couple of more exotic measurements that you can try if you want.

4 Crystal Anisotropy

At room temperature and pressure, both Gallium and Tin crystallize into tetragonal stuctures. The electrical resistivity of single Gallium or Tin crystals will therefore depend on the direction of current flow with respect to the crystal axes.

Figure 8: Gallium crystal structure.

To see the effect of anisotropy, you need a single crystal. One way to grow a large single crystal is to seed a tube of the liquid with a small crystal at the freezing point, and let the liquid slowly cool. The growing solid adopts the orientation of the esd crystal: by orienting the seed, you can determine the orientation of the crystal axes with respect to the boundaries of the liquid's container, resulting in an oriented, single-crystal wire.

The orientation-dependent electrical response of the material depends on the conductivity *tensor* $\sigma(\vec{\mathbf{a}}, \vec{\mathbf{b}})$ according to $\vec{J} = \sigma\vec{E}$. By measuring $\vec{J}$ in the same direction as $\vec{E}$ as

Figure 9: Tin crystal structure.

you have been doing, you are measuring linear combinations of the eigenvalues of $\sigma(\vec{\mathbf{a}}, \vec{\mathbf{b}})$. In a polycrystalline sample, the average over all angles with equal probability yields the customary scalars σ and ρ. In the case of a single crystal, $\rho(\vec{\mathbf{a}}, \vec{\mathbf{b}}$ will be the tensor whose matrix is inverse to σ.

In the absence of strange collective effects, both σ and ρ will share the full symmetry of the crystal. Hence the conductivity and resistivity tensors in tetragonal crystals will have at most six independent components. You can extract these components by measuring the resistivity along various well-defined directions with respect to the axes of the crystal. For more details, I'll refer you to the beautiful reference experiment of R. W. Powell (in the **Gallium References**). Essentially the same experiment can be done on Tin, though because of the Tin's higher melting point, Gallium is probably safer.

One test of your measurements is how closely the angular average of your conductivty tensor (the polycrystalline average) corresponds to the measured σ of polycrystalline Gallium.

5 Observing Phase Transitions

Radical changes in behavior across a well-defined temperature are often a signature of a *phase transition.* A T_{trans} defines the energy scale where the *order parameter*–the physical quantity that defines the character of the phase–stabilizes. Examples of order parameters include the difference in density between a liquid and a gas, the lattice order of a crystalline solid, electric dipole moment of ferroelectrics, the static component of the average electronic spin in magnets, and the collective wave function in superconductors.

Phase transitions depend on collective behavior, wherein the order parameter, communicated from bit to bit of the sample, becomes self-reinforcing enough to withstand fluctuations into competing states. The theoretical "phase transition problem" in condensed matter is thus twofold. It first requires the identification of an order parameter. Then, adding up the feedback to predict a transition temperature constitutes a big counting problem.

5.1 The Liquid-Solid Transition in Gallium

Gallium is a soft metal which melts at about 303 K. You can observe its phase transition from solid to liquid outside of the refrigerator by placing the Gallium sample on a thermoelectric pad in thermal contact with a calibrated thermometer. At the melting point temperature, you should observe a discontinuity between the conductivities of the liquid and the solid.

5.2 Structural Transitions in Tin

Structural changes affect the band structure of the allowed electronic states, and so can have a strong signature in the conductivity. In terms of the Drude formulation, the band structure directly determines n; and the crystal structure plays the major role in determining the band structure. One such transition famously occurs in Tin at 286 K. Above this T_{struct}, Tin is a ductile metal ("white tin"); at temperatures below 286 K, its crystal structure changes and it becomes a brittle insulator ("grey tin" or "tin pest"). See if you can locate the signature of this transistion in σ (you may use the flip side of the thermoelectric pad to cool the tin).

5.3 Magnetism

You are certainly familiar with bulk ferromagnetism, the magnetism of refrigerator magnets. The ferromagnetic order parameter–the static part of the magnetic moment of the conduction electrons–vanishes at temperatures greater than about 1040K (770C). Many other materials exhibit magnetic ordering, often at much lower temperatures–and the magnetic order does not necessarily lead to a field that can be seen outside of the sample. Nevertheless, the magnetic moments of the electrons in the sample are interacting strongly enough that they line up in various kinds of arrangement. When, as is often the case, the magnetic moment is attached to the conduction charge carriers, the magnetic order may have echoes in σ.

5.4 The Magnetic Susceptibility χ_M

A more telling sign of a magnetic phase transition is a change in the *magnetic susceptibility* χ_M. To put χ_M in context, if σ determines the response relating charged currents to applied electric field, then χ_M governs the response of the magnetic moments (often, the electron spins) to an applied magnetic field. When the moments are not interacting strongly with each other, they respond more easily to an applied magnetic field. You might say that the individual moments behave analogously to the free atoms in a gas, and are relatively easy to flip around because they move independently. When the interaction amongst the moments becomes important, however, the motion becomes more "viscous," and eventually, if the moments "freeze," then it becomes difficult to rotate them arbitrarily.

Figure 10: Magnetism Cartoon.

If a magnetic sample is placed inside the coil of an LRC circuit, the change in response below the magnetic transition temperature T_M will change the effective inductance of the circuit and hence change the resonance frequency curve. In practice, one often uses a two-coil technique which is the magnetic analogue of the four-wire resistance measurement: a *drive coil* puts out a changing flux, while the voltage measured across the terminals of the second, *search* coil with the sample inside or just behind it measures how much gets through.

To measure the frequency-dependence of χ_M, you will use a special device known as a *lock-in amplifier*. A susceptibility implies a correlation between the frequency of the excitation and the frequency of the response. The lock-in method depends on this correlation to amplify weak signals at a selected frequency–the correlation allows for enhanced noise rejection by averaging the total signal over time against a definite sine wave.

You are able to adjust the frequency at which you drive, the frequency at which you search, and the relative phase of the two signals. The phase adjustability allows you to look at the real and imaginary parts of χ_M separately.

Why would you want to look at other response frequencies? If the magnetic response of the material were linear with no dissipation, you would expect a large spike at precisely the excitation frequency. Nonlinearities lead to response at harmonics, while dissipation of the energy put into the excitation broadens the response from a spike to a peak of finite width.
For the fine details of running the lockin, consult the **Lockin Notes**.

Figure 11: Lock-In Processing Cartoon.

5.5 Superconductivity

While the idea of perfect metallic conductivity is understandable within the context of the Drude model for independent electrons, *superconductivity* (which by contrast is an actually attainable state) is not. While normal currents in a metal decay by scattering, for supercurrents there is an energy gap–analogous to the band-gap that makes insulators work–which means that scattering itself requires energetic activation. Hence materials in the superconducting state are qualitatively different from merely excellent regular conductors.

One manifestation of this qualitative difference is the anomalous response of the superconductor to magnetic fields (this is called the *Meissner effect*). To understand its significance, first consider the theoretical response of a perfect metal to a changing magnetic field. According to Faraday's law of induction, the changing magnetic field would induce current loops that oppose the change in the field. Hence the field in a perfect conductor would be stuck at its initial value. On the other hand, a superconductor tends either to expel (type I) or sequester (type II) magnetic field lines so that most of the sam-

Figure 12: SC Resistive Transition.

ple volume is free of magnetic field. A superconductor spontaneously generates electric currents that screen fields from the bulk of the sample.

Figure 13: Meissner Cartoon.

How does this energy-gapped, magnetophobic state arise? Superconductivity requires a complicated choreography of the conduction electrons. In one fairly well-understood class of superconductors (Bardeen-Cooper-Schreiffer or BCS-type), the motions of the electrons become coordinated by sharing the energy benefit of lattice deformations. The overlap of the coordinated electron wavefunctions leads to a bulk coherent state–a macroscopic quantum mechanical phenomenon!–which is energetically favorable below the superconducting transition temperature T_{SC}. Like bulk magnetism, superconductivity is *a different thermodynamic state of solid matter*, due to quantum mechanical interactions. For more about the superconducting state, see the **Superconductivity References**.

Superconductivity was first discovered in solid mercury by Heike Kammerleigh Onnes in 1911, soon after his lab had succeeded in the liquefaction of helium. Superconducting wires with the ability to carry large currents are used in the production of the very strong cryogenic electromagnets used in diagnostic NMR machines. In the 1980's, Bednorz and Muller discovered superconductivity at the relatively high temperature of 35 K in a layered copper-oxide. As part of this lab, you have the opportunity to make (or, if time doesn't permit, at least measure) a similarly structured compound that becomes supercondcuting anywhere from $70-90$ K, depending on the details of how you produce it (see the notes for **Making BSCCO**). Although the recipe for making this material is well known, there is as yet no consensus about the interaction which makes its superconducting transition occur.

To confirm that a sample is superconducting, you must do two things. You must certainly show that the sample conducts electricity without resistance with a four-wire measurement. Furthermore, you must establish the anomalous response to a magnetic field. For the BSCCO samples, which are high-temperature superconductors, you can see whether they expel a magnetic field by repelling a small NdBFe magnet, or change the local pattern of scattered iron filings in an applied field, when they are cooled to liquid nitrogen temperature in an open dewar. If there are MgB_2, Nb, or other superconducting samples with T_{SC} lower than 77 K around, you would need to check their resistive and magnetic transitions in the refrigerator.