AMERICAN JOURNAL OF PHYSICS VOLUME 36, NUMBER 8 AUGUST 1968

Student Laboratory Demonstration of Flux Quantization and the Josephson Effect in Superconductors

P. L. RICHARDS
Department of Physics and Inorganic Materials Research Division, Lawrence Radiation Laboratory, University of California, Berkeley, California 94720
SIDNEY SHAPIRO
Department of Electrical Engineering, University of Rochester, Rochester, New York 14627
AND
C. C. GRIMES
Bell Telephone Laboratories, Murray Hill, New Jersey 07971
(Received 11 March 1968)

A simple apparatus is described which permits observation of magnetic flux quantization and the alternating current Josephson effect in superconductors. The apparatus, which has been used successfully by students in the Advanced Undergraduate Physics Laboratory at Berkeley, consists of point contacts between two niobium wires which are attached to a probe so that the contact pressure can be adjusted while the contacts are in a storage Dewar of liquid helium. The current-voltage characteristic of the point contact junction is displayed on an oscilloscope. Flux quantization causes the zero-voltage current carried by the junction to oscillate as a magnetic field is applied by moving a permanent magnet close to the storage vessel. The ac Josephson effect causes constant voltage steps to appear on the current-voltage characteristic when a small amount of power from a K-band ($\sim$24-GHz) klystron is coupled to the points.

INTRODUCTION

The phenomena of superconductivity are of great interest to the student of physics. They are sufficiently far removed from everyday experience that laboratory experiments are desirable. This is especially true of the recently discovered macroscopic quantum phenomena assoicated with the superconducting state: flux quantization and the Josephson effect. These phenomena provide a unique bridge between the microscopic quantum "world" and the macroscopic world of everyday experience. They provide, for example, a method for measuring Planck's constant with a current source, a bar magnet, a voltmeter, and a few bits of superconducting wire!

The theoretical background for understanding many of these phenomena has been made available to the undergraduate who has some understanding of basic quantum mechanics by the discussion of the Josephson effect in Vol. III of *The Feynman Lectures on Physics*.[1] Feynman gives a brief description of several Josephson effect experiments and their quantum mechanical explanation as his final example of how "We are really getting control of nature on a very delicate and beautiful level."

Because of their fundamental importance and because of their great capacity to interest students of physics, it is desirable to offer a Josephson effect experiment in a senior or first-year graduate laboratory course. The difficulties involved in doing so may, at first, seem overwhelming. Of course, such experiments are limited to institutions in which liquid helium is available. Most universities and an increasing number of colleges do use liquid helium. The second difficulty is the preparation of specimens. Josephson effect experiments are usually done with evaporated film tunnel junctions whose preparation is a difficult and often unreliable process far beyond the capabilities of the usual student laboratory.

In this paper we describe simple apparatus which can be used to demonstrate flux quantization and the ac Josephson effect in a student laboratory. Our method uses easily fabricated point contacts rather than evaporated film junctions, and experiments can be done directly in a liquid-helium storage Dewar. Thus, the use of expensive cryostats and the wasteful transfer of liquid helium are avoided.

[1] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley Publ. Co., Reading, Mass., 1965), Vol. 3, Chap. 21.

THEORY

Flux Quantization

The theory of flux quantization and the Josephson effect has been presented in a variety of ways.[1–3] Of most use to the undergraduate student are Feynman's discussion[1] and an article by Langenberg, Scalapino, and Taylor in *Scientific American*.[3] We present here only one possible way of approaching the subject.

The wave function, or order parameter, for the superconducting state is known to have the simple form $\psi=\psi_0 e^{i\theta}$, where the phase factor θ is a function of both position and time. Since ψ is required to be single valued,

$$\oint \nabla\theta\cdot d\mathbf{l}=2\pi n, \qquad (1)$$

where $n=0, 1, 2\cdots$, and the line integral is taken around a closed path entirely within the superconductor. For a wave function of this form, the usual quantum-mechanical expression for current density is[1]

$$\mathbf{J}=(\hbar/m)[\nabla\theta-(q/c\hbar)\mathbf{A}]\,|\psi_0|^2, \qquad (2)$$

where $\mathbf{A}$ is the magnetic vector potential and $|\psi_0|^2$ is the density of superconducting electrons. Using this value for $\nabla\theta$ in Eq. (1), we get

$$\oint\left(\frac{mc\mathbf{J}}{q\,|\psi_0|^2}+\mathbf{A}\right)\cdot dl=\frac{nhc}{q}.$$

We can, of course, write

$$\oint \mathbf{A}\cdot d\mathbf{l}$$

as the magnetic flux

$$\int_s \mathbf{B}\cdot d\mathbf{s}$$

threading the path of integration. These arguments, which use the quantum mechanics of single particles, are modified by the interaction between pairs of electrons in a superconductor so that the effective charge q is $2e$, twice the free electron charge. Thus, we finally obtain

$$\int_s \mathbf{B}\cdot d\mathbf{s}+\frac{mc}{2e\,|\psi_0|^2}\oint \mathbf{J}\cdot d\mathbf{l}=\frac{nhc}{2e}, \qquad (3)$$

where the path of integration, but not necessarily the surface spanning it, lies entirely within the superconductor. For a thick superconducting ring, it is possible to choose a path in a current free region deep within the superconductor where the field and current do not penetrate. Equation (3) then shows that the flux threading the ring is quantized in units of $hc/2e$. The full quantized quantity including the current term is called a fluxoid. If the path of integration comes close to the surface, both the current and field terms depend on the path in such a way that the fluxoid is conserved.

It should be noticed that the prediction of fluxoid quantization is valid only if the phase θ of the superconducting wave function is coherent over distances at least as large as the perimeter of the loop. The observation of such effects in loops having perimeters of a meter or more[4] demonstrates the existence of quantum effects extending over macroscopic distances.

Perhaps the easiest way to verify Eq. (3) experimentally is to use the superconducting circuit shown in Fig. 1(a). Superconducting electrons can pass through the thin oxide barrier by quantum mechanical tunneling (the direct-current Josephson effect). There is, however, a relatively low maximum lossless current I_0 in the tunnel junctions above which a voltage drop appears. We can now ask the question: How will a small externally imposed current I divide itself between the two branches of this lossless circuit? If the current divides itself equally so that

$$\oint \mathbf{J}\cdot d\mathbf{l}=0$$

around the loop, a maximum zero voltage current $I=2I_0$ can flow across the circuit. According to Eq. (3), this can only occur when the flux threading the circuit equals $nhc/2e$, where $n=0, \pm1, \pm2\cdots$. If the flux has an intermediate value, say $(n+\frac{1}{2})hc/2e$, then there will be a finite

$$\oint \mathbf{J}\cdot d\mathbf{l}$$

[2] B. D. Josephson, Phys. Letters **1**, 251 (1962) and Advan. Phys. **14**, 419 (1965).

[3] D. N. Langenberg, D. J. Scalapino, and B. N. Taylor, Sci. Am. **214**, 30 (1966).

[4] L. L. Vant-Hull and J. E. Mercereau, Phys. Rev. Letters **17**, 629 (1966).

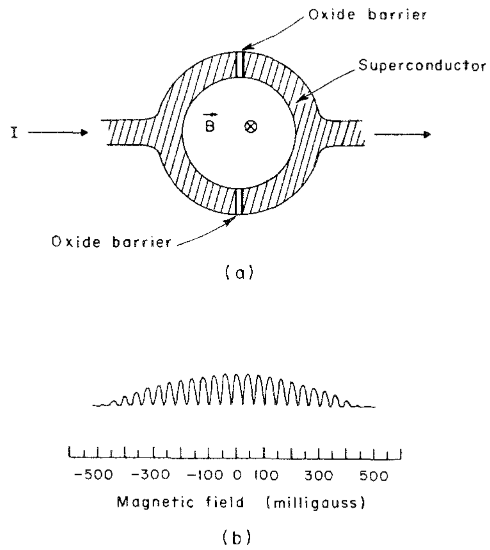

FIG. 1 (a) Schematic representation of a superconducting double-juction circuit. (b) Plot of the maximum zero voltage current vs magnetic field for the circuit of Fig. 1(a) from the work of Jaklevic, Lambe, Mercereau, and Silver (Ref. 5).

and this circulating current will cause resistance to appear for $I<2I_0$. Thus, the maximum zero-voltage current that can be carried by the circuit is an oscillatory function of the applied magnetic field as is shown in Fig. 1(b). A more careful analysis shows that the theoretical prediction for Fig. 1(b) is the modulus of the amplitude of a two-slit diffraction pattern.[5] In general, for n tunnel junctions connected in parallel, the maximum zero-voltage current plotted as a function of magnetic field has the form of an n-slit diffraction pattern.

Our demonstration of flux quantization in superconductors consists of the observation of this field induced oscillation of the zero-voltage current in a multiple tunnel junction circuit.

The Alternating Current Josephson Effect

Quantum-mechanical tunneling between two superconductors has been studied extensively. The subject can be divided into two parts: single particle tunneling and Josephson tunneling. The first to be discovered was the tunneling of individual electrons, or quasiparticles, which dominates the current for junctions with relatively thick oxide barriers when a finite voltage is applied. This effect has been used extensively for the study of the energy gap in the density of states for excitations from the superconducting ground state.[6] Josephson showed that correlated pairs of electrons will also tunnel and should allow current flow with no voltage drop in thin junctions. His calculations showed that this lossless pair tunneling or Josephson current depends on $\Delta\theta$, the phase difference between the wave functions for the superconductors on the two sides of the junction[1,2]:

$$I=I_0 \sin\Delta\theta. \tag{4}$$

Here I_0 is the maximum zero-voltage current which can be carried by the junction. The two junction experiments described above can also be analyzed in terms of the influence of the magnetic field on $\Delta\theta$ for the two junctions.[1,5]

We now consider the effect of the finite potential drop V across a single barrier which occurs when I_0 is exceeded. Josephson[2] showed that this leads to a time dependence of the phase difference

$$\hbar(d/dt)(\Delta\theta)=2eV_0. \tag{5}$$

This result for superconductors is analogous to that obtained from the elementary quantum mechanics of a single particle system.[7] If the voltage V_0 is constant in time, Eq. (5) can be integrated to give $\Delta\theta=2eV_0t/\hbar+C$. Equation (4) then predicts an alternating current flow across the barrier at the frequency $\omega=2eV_0/\hbar$. This is a high frequency under most circumstances since $\omega/2\pi=484$ GHz/mV. The existence of this Josephson alternating current has been verified by observing the microwave radiation emitted at the frequency ω.[3] The ac Josephson currents

[5] See J. M. Rowell, Phys. Rev. Letters **11**, 200 (1963) for a single junction and R. C. Jaklevic, J. Lambe, J. E. Mercereau, and A. H. Silver, Phys. Rev. **140**, A1628 (1965) for two junctions in parallel.

[6] C. Kittel, *Introduction to Solid State Physics* (John Wiley & Sons, Inc., New York, 1966), 3rd ed., pp. 361–363.

[7] Assuming a wave function of the general form $\psi=\psi_0e^{i\theta}$ and Schrödinger's equation $-i\hbar\dot{\psi}=\mathcal{H}\psi$, the time dependence of the phase difference $\Delta\theta$ between two states is $\hbar d(\Delta\theta)/dt=\Delta E$. Josephson showed that in the superconducting tunnel junction, the change in system energy $2eV_0$ when a superconducting pair moves from one side of the barrier to the other plays the role of ΔE, the energy difference between the two single particle states.

were first detected, however, by an easier experiment.[8] An alternating voltage $V_1 \cos\omega_1 t$ at a microwave frequency ω_1 was induced across the barrier in addition to the steady voltage V_0. Equation (5) then yields

$$\Delta\theta = (2eV_0 t/\hbar) + (2eV_1 \sin\omega_1 t/\hbar\omega_1) + C,$$

so from Eq. (4)

$$I = I_0 \sin[(2eV_0 t/\hbar) + (2eV_1/\hbar\omega_1) \sin\omega_1 t + C]. \quad (6)$$

This equation shows that the frequency of I is modulated by the microwave frequency ω_1. It is usual to expand such expressions using the relations[9]

$$\cos(X \sin\theta) = \sum_{n=-\infty}^{\infty} J_n(X) \cos n\theta$$

and

$$\sin(X \sin\theta) = \sum_{n=-\infty}^{\infty} J_n(X) \sin n\theta,$$

where $J_n(X) = (-1)^n J_{-n}(X)$ is the Bessel function of order n. Choosing the value $C = \pi/2$ which corresponds to a maximum zero-voltage $(V_0 = V_1 = 0)$ current and using standard trigonometric identities, we obtain

$$I = I_0 \sum_{n=0}^{\infty} J_n\left(\frac{2eV_1}{\hbar\omega_1}\right) \times [\cos(\omega + n\omega_1)t + (-1)^n \cos(\omega - n\omega_1)t]. \quad (7)$$

The junction is a nonlinear device in which the ac Josephson currents beat with the microwave currents. Beats at zero frequency appear whenever $\omega \pm n\omega_1 = 0$. These can be observed by measuring the dc current-voltage characteristic of the junction. A lossless contribution to the dc current appears whenever the voltage is adjusted so that the ac Josephson frequency $\omega = 2eV_0/\hbar$ equals harmonics of the microwave frequency ω_1. The amplitude of this dc beat current is given by the Bessel functions $J_n(2eV_1/\hbar\omega_1)$, where n is the order of the harmonic. The four lowest order Bessel functions are plotted in Fig. 2.

[8] S. Shapiro, Phys. Rev. Letters **11,** 80 (1963), and S. Shapiro, A. R. Janus, and S. Holly, Rev. Mod. Phys. **36,** 223 (1964).

[9] W. G. Bickley, *Bessel Functions and Formulae* (Cambridge University Press, Cambridge, England 1957), Formulas 94(a) and 94(b).

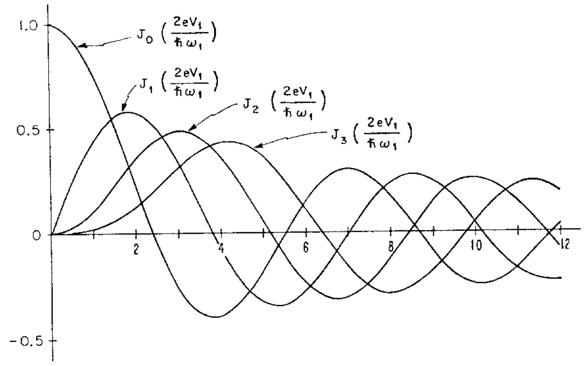

FIG. 2. Plot of the Bessel functions J_0, J_1, J_2 and J_3.

The observation of constant voltage steps on the I–V characteristic of a Josephson junction placed in a microwave field thus verifies the existence of the ac Josephson currents. This experiment has considerable practical importance. Careful measurements of the applied microwave frequency and the voltage at which the steps occur have recently yielded the best numerical value for the ratio of fundamental constants e/h.[10] Also, a Josephson junction can be used in this mode as an extremely sensitive detector of microwave and far-infrared radiation.[11]

APPARATUS

In order to offer the experiment described in the previous section in a student laboratory, an easily constructed tunnel junction is required. This need is well satisfied by point-contact Josephson junctions made from niobium (Nb) wire. A small (5- to 30-mil)-diam wire of Nb should be sharpened to a point of a few mm radius with a file or sandpaper and then allowed to remain undisturbed for a day or two while an oxide layer forms. When such a point is pressed against a flat Nb surface with ≈1 oz of force, the oxide layer is deformed and a Josephson tunneling junction can be obtained. Figure 3 shows the apparatus used in the Berkeley Advanced Undergraduate Physics Laboratory for adjusting the contact pressure while the points are immersed in a liquid-helium storage Dewar. Several extra

[10] W. H. Parker, B. N. Taylor, and D. N. Langenberg, Phys. Rev. Letters **18,** 287 (1967).

[11] C. C. Grimes, P. L. Richards, and S. Shapiro, Phys. Rev. Letters **17,** 431 (1966), and J. Appl. Phys. (to be published).

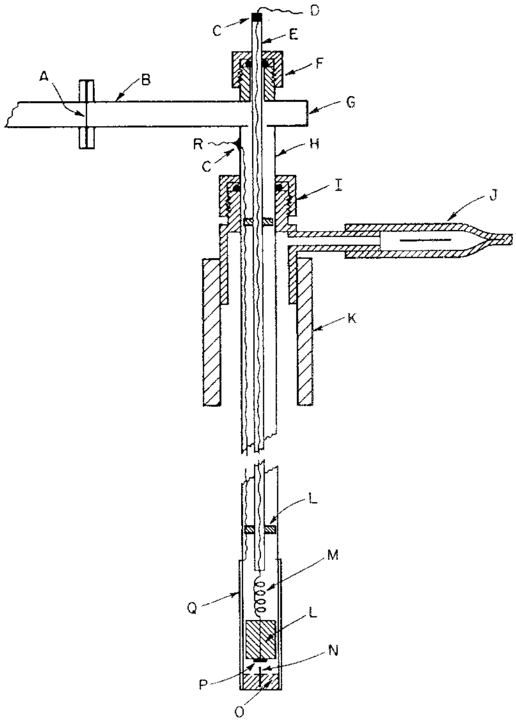

FIG. 3. Point contact probe for use in helium storage Dewar. (A) Polyethylene seal. (B) *K*-band waveguide. (C) Epoxy seal. (D) Copper potential measuring lead. (E) Ten-mil wall stainless tube used for adjusting contact force. Stainless steel was chosen for its small thermal conductance. (F) O-ring gland. (G) Termination of guide is not critical. (H) $\frac{1}{2}$-in.-o.d. 20-mil wall stainless tube. (I) Adjustable collar for sealing neck of storage Dewar. (J) Bunsen valve to relieve gas pressure in Dewar. (K) Rubber tube to fit neck of storage Dewar. (L) Insulating spacer. (M) $\sim\frac{1}{2}$ in. oz spring. (N) Niobium point. (O) Brass plug. (P) Niobium flat. (Q) Unless 316 or 321 stainless steel is used, the lower few inches of probe should be made of brass to avoid ferromagnetism of most stainless steels at low temperatures. (R) Copper potential measuring lead. The current supply contacts to the probe are made through the stainless tubes *E* and *H*.

points should be kept on hand so that a new one can be easily inserted if the oxide layer is damaged by overvigorous adjustment.

The details of the probe design are not critical. The following requirements should be kept in mind. The contact force should be adjustable smoothly. Separate current and potential leads should be provided for measuring the *I–V* characteristic. The potential leads should be of the same material to avoid difficulty from a thermal electromotive force. We have used Teflon-insulated copper wire. The top of the storage Dewar and the probe should be sealed to prevent the entry of air which would freeze in the storage Dewar during the course of the experiment. Provisions must be made, however, to release the evolved He gas. A slit in a rubber tube (Bunsen valve) is adequate. It is also necessary to couple *K*-band or higher frequency microwaves down the probe which can be considered to be a very nearly shorted co-axial line. Since small amounts of power are needed at the point contacts, the coupling is not at all critical.

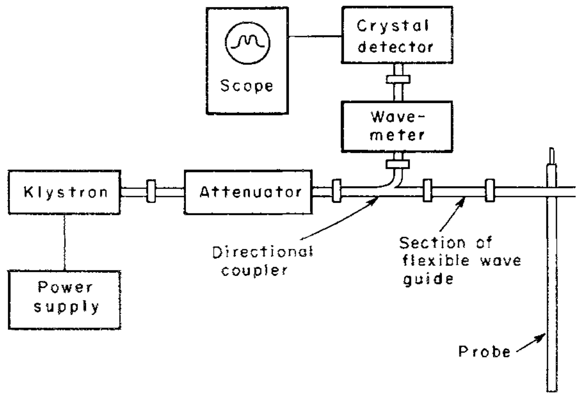

FIG. 4. Schematic diagram of microwave system.

Figure 4 shows the simple microwave circuit used for our experiments. The klystron is a surplus 2K33 which radiates $\approx$10 mW in the vicinity of 24 GHz. Figure 5 shows an inexpensive power supply for the 2K33.[12] By using surplus components whenever possible, it was constructed for $\approx$\$40.

Figure 6 shows an adequate circuit for sweeping the *I–V* characteristic of the junction on an oscilloscope. The ideal scope for this purpose is a Tektronix 502A or a 536 with two 1A7 plug-ins. Such elaborate equipment is not necessary, however. Some of our experiments have been done with an ancient DuMont scope and two triode (12AX7) preamplifiers. A sensitivity of $\approx$100 uv/cm on the voltage axis and preferably at least 1 mV/cm on the current axis is required. The bandwidth should extend to about 100 kHz on the voltage axis to resolve the sharp corners on the *I–V* curves.

[12] We are indebted to R. La Forge for the design and construction of this power supply.

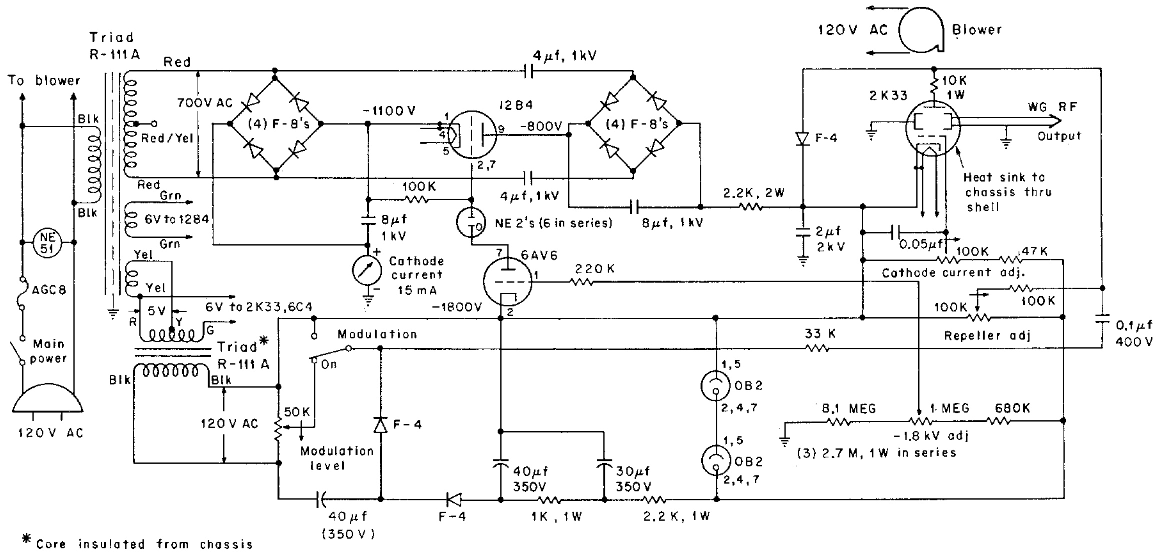

FIG. 5. Schematic diagram of klystron power supply. Supply uses 60 cps sine-wave frequency modulation for display of klystron mode on scope.

EXPERIMENTAL PROCEDURE

Flux Quantization

The observation of flux quantization is made by inserting the probe into the helium and adjusting the contacts until an I–V curve is seen similar to that shown in Fig. 7(a) or (b). Notice that far from the origin the inverse slope of the I–V curve corresponds to a resistance of a few ohms. The vertical line centered about the origin is the zero-voltage (superconducting) current carried by the junction. It is generally made up from both superconducting flow through a direct metallic contact and dc Josephson current tunneling through oxide barriers. Since the points are rough on a microscopic scale, we usually have several tunnel junctions in parallel with very small included area. Thus, if a magnetic field of a few gauss is applied to the point contact, that portion of the zero-voltage current due to dc Josephson tunneling will oscillate as a several slit diffraction pattern. This is easily observed

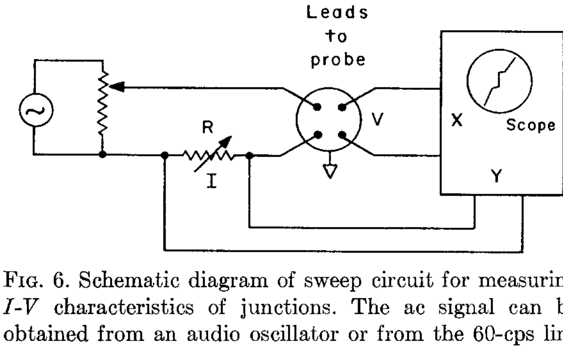

FIG. 6. Schematic diagram of sweep circuit for measuring I-V characteristics of junctions. The ac signal can be obtained from an audio oscillator or from the 60-cps line voltage stepped down through a transformer. Useful values of R lie in the range $1 < R < 500\Omega$.

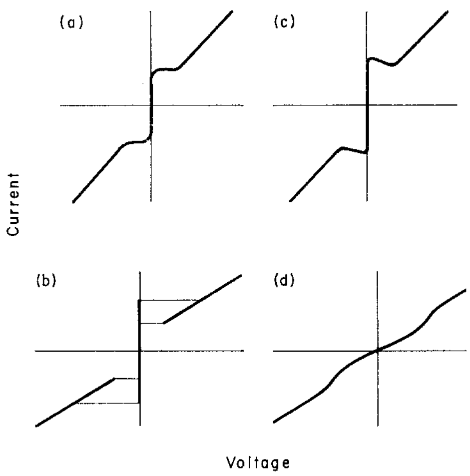

FIG. 7. Tracings from scope photos showing various forms of I-V characteristics of Nb point contact tunnel junctions at 4.2°K. Figures 7(a), 7(b) and especially 7(c) are characteristic of point contacts with Josephson tunneling. Figure 7(d) shows a large contribution from single particle tunneling. The normal state junction resistances obtained from the slope far from the origin were 10, 4, 40, and 500 Ω for 7(a) through 7(d), respectively.

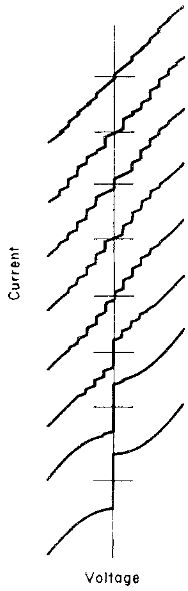

FIG. 8. Current voltage characteristic of a Nb point contact at 4.2°K showing constant voltage steps induced by *K*-band microwave radiation at 23.5 Gc/sec. The lowest curve is for zero power and each higher curve corresponds to increased incident power. The junction resistance was ≈4 Ω and the amplitude of the voltage sweep was 0.38 mV.

on the oscilloscope when a fairly large *C* magnet is moved close to the storage Dewar. Occasionally the oscillations will not be seen. This is usually due to an overvigorous adjustment of the points which can damage the oxide barrier and cause the dominant portion of the current to be carried by direct metallic contact. When no field dependence is seen after several tries, another set of points, prepared in advance and allowed to oxidize, should be installed.

The experiment described here is a qualitative verification of flux quantization. Quantitative measurements are not possible because neither the area included by the junction nor the magnetic field penetrating the (usually ferromagnetic) storage vessel is known. Quantitative experiments are not at all difficult to design, but usually require more space than is allowed by the neck of the storage Dewar. Several useful, easily constructed double-point contact structures which are equivalent to Fig. 1(a) are described by Zimmerman and Silver.[13] Another interesting form of double junction highly suitable for student laboratory experiments has been discussed by Clarke.[14] It can also be used to observe the ac Josephson effect.

[13] J. E. Zimmerman and A. H. Silver, Phys. Rev. **141**, 367 (1966).

[14] J. Clarke, Phil. Mag. **13**, 115 (1966).

A wide variety of *I–V* characteristics can be obtained by a suitable adjustment of the point contacts. Some typical examples taken from photographs of the oscilloscope display are shown in Fig. 7(a) through (d). Figure 7(b) shows hysteresis which is often observed when the current measuring resistor *R* is large. Figure 7(c) shows a region of negative differential resistance which only appears on the scope if the impedance of the current source is low—i.e. if the load line is steep.[15] Occasionally, when they are biased in this region, point contacts will function as a negative resistance oscillator. The frequency of oscillation, which can be estimated from the "hash" on the negative resistance region, is often found to be a few kHz. It is probably determined by external circuit parameters.

It is possible, though more difficult, to observe *I–V* characteristics such as the one shown in Fig. 7(d). Levinstein and Lunzler[16] have anodized points to produce a thick oxide barrier. The Josephson tunneling is then suppressed and current flows due to single particle tunneling but not until a voltage $eV \simeq 3.5kT_c$ equal to the super-

[15] This negative resistance region can be understood as follows. When the voltage is zero the dc Josephson current contributes to the total dc current flow. At finite voltages, however, the Josephson current occurs at high frequencies (484 GHz/mV) and thus does not contribute to the current seen on the scope. Thus the Josephson contribution to the measured current vanishes when a finite voltage is applied. Of course, the net current measured cannot decrease more rapidly with voltage than the slope of the load line or the system will be unstable and the scope trace will jump rapidly across the negative resistance region. This behavior is often seen as a faint horizontal line connecting the maximum zero voltage current to the positive resistance region at a few millivolts as in Fig. 7(b). Whether the net current flow actually shows a *negative* resistance depends on the fraction of current carried by direct metallic contacts. This contribution to the current has a positive resistance in the low voltage region due to the phenomenon of flux flow [Y. B. Kim, C. F. Hempstead, and A. R. Strnad, Phys. Rev. **139**, A1163 (1965)]. Thus the low voltage or "horizontal" portion of the *I-V* trace which connects the maximum zero-voltage current to the region of normal state resistance at high voltage can show positive or negative resistance or even instability depending on the experimental conditions. Examples of each of these types of behavior are shown in Figs. 7(a), 7(c), and the zero microwave power (lowest) curve in Fig. 8.

[16] H. J. Levinstein and J. E. Kunzler, Phys. Letters **20**, 581 (1966).

conducting energy gap is exceeded.[6] In Fig. 7(d) a bump is seen corresponding to the onset of single particle tunneling at $\approx$2.2 mV. This value is somewhat smaller than the gap voltage of $\approx$2.9 mV reported for pure Nb under ideal conditions. This type of experiment is the classic method for measuring the density of states for excitations in a superconductor.[6] It should be kept in mind that single particle tunneling contributes to the current flow for voltages greater than the gap in all of the I–V curves shown in Fig. 7.

The Alternating Current Josephson Effect

When an I–V curve is obtained in which the zero-voltage current oscillates with magnetic field, that is, one with a significant amount of Josephson current, the application of a microwave frequency voltage will generally produce steps on the I–V curve similar to those shown in Fig. 8. Such steps have been observed at a wide variety of frequencies, but are most easily seen at K-band or higher frequencies. The vertical edge of each step is a contribution to the dc current given by Eq. (7) when the voltage is such that the ac Josephson frequency is a harmonic of the microwave frequency. Equation (7) actually predicts current spikes at these values of voltage. If the I–V curve were measured with a vertical load line (constant voltage source) then such spikes would be experimentally observed. In practice, however, such an experiment is rather difficult. The current measuring resistor R in Fig. 6 is generally much larger than the junction resistance so that the load line is nearly horizontal. This produces the steps shown in Fig. 8. Calculated I–V curves for ac Josephson experiments with vertical and horizontal load lines are given by Werthamer and Shapiro.[17] Since the current is more nearly the independent variable in these measurements, some authors plot V–I curves with current on the ordinate rather than the I–V curves used here.

For many point contacts it will be noticed that steps occur more frequently than expected from Eq. (7). This usually means that a large fraction of the current is being carried by direct metallic contact. This situation has been investigated by measuring the I–V characteristic of a narrow neck of a superconductor in a microwave field.[18] Steps are then expected and observed when the Josephson frequency is either a harmonic *or a subharmonic* of the microwave frequency. That is, when $V_0 = n\hbar\omega_1/n'2e$ where n and $n' = 1, 2, 3\cdots$.

The ac Josephson experiment is quantitative in the sense that the microwave frequency can be measured with a cavity wavemeter and the voltage of the steps measured from the scope to obtain a value for e/h. A further test of the functional form of Eq. (7) can be made by observing the Bessel function dependence $J_n(2eV_1/\hbar\omega_1)$ of the height of the nth step on the microwave voltage. A calibrated microwave attenuator can be used, as shown in Fig. 4, to obtain quantitative data.[8] During the course of these experiments, it is noticed that a very small amount of microwave power drastically reduces the zero-voltage current. This effect has been used as a sensitive microwave and far-infrared detector.[11] Also, with large amounts of microwave power, all evidence of superconductivity is obliterated. It is very helpful if a scope camera is available to record some of these observations.

CONCLUSIONS

We have described apparatus which permit simple measurements of several of the macroscopic quantum phenomena recently discovered in superconductors. These experiments have been found to be of considerable interest to students in the Berkeley Advanced Undergraduate Physics Laboratory. During the experiments it is sometimes necessary for the seniors to borrow microwave equipment and liquid helium from research groups. This interaction, and the interest shown in these experiments by graduate students has helped to break down some of the artificial barriers between the teaching and research laboratories.

ACKNOWLEDGMENTS

The authors are indebted to G. Adams for taking the pictures shown in Figs. 7 and 8. A portion of this work was performed under the auspices of the U. S. Atomic Energy Commission.

---

[17] N. R. Werthamer and S. Shapiro, Phys. Rev. **164**, 523 (1967).

[18] A. H. Dayem and J. J. Wiegand, Phys. Rev. **155**, 419 (1967).